PROBLEM SET 8 DATA STRUCTURES AND ALGORITHMS 2 - FALL 2024

your name here

DUE NOVEMBER 13, 2024 AT 11:59P

Collaboration Policy: You are encouraged to collaborate with up to 4 other students, but all work submitted must be your own *independently* written solution. List the computing ids of all of your collaborators in the collabs command at the top of the tex file. Do not share written notes, documents (including Google docs, Overleaf docs, discussion notes, PDFs), or code. Do not seek published or online solutions for any assignments. If you use any published or online resources (which may not include solutions) when completing this assignment, be sure to cite them. Do not submit a solution that you are unable to explain orally to a member of the course staff. Any solutions that share similar text/code will be considered in breach of this policy. Please refer to the syllabus for a complete description of the collaboration policy.

Collaborators: list your collaborators **Sources**: list your sources

PROBLEM 1 Arithmetic Optimization

You are given an arithmetic expression containing *n* integers and the only operations are additions (+) and subtractions (-). There are no parenthesis in the expression. For example, the expression might be: 1 + 2 - 3 - 4 - 5 + 6.

You can change the value of the expression by choosing the best order of operations, for example:

$$((((1+2)-3)-4)-5)+6 = -3) \\ (((1+2)-3)-4)-(5+6) = -15) \\ ((1+2)-((3-4)-5))+6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + 6 = 15) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-((3-4)-5)) + ((1+2)-((3-4)-5)) \\ ((1+2)-(($$

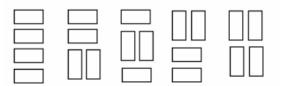
Give a **dynamic programming** algorithm that computes the maximum possible value of the expression. You may assume that the input consists of two arrays: nums which is the list of n integers and ops which is the list of operations (each entry in ops is either '+' or '-'), where ops[0] is the operation between nums[0] and nums[1]. Your answer should include showing expressions from which the min and/or max value is taken, as shown on the slides for the DP solutions that we've covered. Also describe how the intermediate values are stored and where in that memory structure the final answer (maximum possible value) will be. You do not need explain how the optimal ordering can be extracted after the final answer is computed. *Hint: consider a similar strategy to our algorithm for matrix chaining*.

Solution:

PROBLEM 2 More Dominos

In class, we considered the following problem: suppose you are given a rectangular board of size 2 by $w \ge 1$ (the units here don't matter). You are also given an endless bag of dominoes, each of size 2 by 1. Write an algorithm that accepts the integer w as a parameter and returns the total number of unique ways the 2 by w board can be fully tiled.

Now, for homework, solve the exact same problem as before, except with a board of size 4 by w. Describe your algorithm and state its runtime. For example, if the input given is w = 2, then the solution is 5, as shown in the following image (all of the combinations of fully tiled boards of width w = 2):



Solution: