Binary Arithmetic, Bitwise Operations

CS 2130: Computer Systems and Organization 1
January 21, 2026

Announcements

- My Office Hours
  - Tuesdays 11:00am - 12:00pm
  - Wednesdays 2:00pm - 3:30pm
- TA Office Hours on website (location coming soon)
- Homework 1 available later this week

Multi-bit Values

- So far, only talking about 2 things
- Numbers, strings, objects, …

Numbers

From our oldest cultures, how do we mark numbers?

- **unary** representation: make marks, one per ”thing”
  - Awkward for large numbers, ex: CS 2130?
  - Hard to tell how many marks there are
- Update: group them!
- Romans used new symbols: V, X, L, C, M

Numbers

From our oldest cultures, how do we mark numbers?

- Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - Positional numbering system

7

10^n

2 1 3 0

10,000s 1000s 100s 10s 1

$\ldots \; 10^4 \quad 10^3 \quad 10^2 \quad 10^1 \quad 10^0$

$\approx 2 \times \underset{10^3}{1000} + 1 \times \underset{10^2}{100} + 3 \times 10$

Numbers

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  - Positional numbering system
  - The 10 is significant:
    - 10 symbols, using 10 as base of exponent

Numbers

From our oldest cultures, how do we mark numbers?

- Arabic numerals
  - Positional numbering system
  - The 10 is significant:
    - 10 symbols, using 10 as base of exponent
  - The 10 is *arbitrary*
  - We can use other bases! π, 2130, 2, ...

Base-8 Example

Try to turn 134_8 into base-10:

0, ... 7

$1 \times 64 + \underbrace{3 \times 8}_{24} + 4 \times 1$

...			1	3	4	=				9	2
	8^4	8^3	$8^2 = 64$	$8^1 = 8$	$8^0 = 1$		$10^4 = 10000$	$10^3 = 1000$	$10^2 = 100$	$10^1 = 10$	$10^0 = 1$

10

$10 \rightarrow 8$

Bases

We will discuss a few in this class

- Base-10 (decimal) - talking to humans
- Base-8 (octal) - shows up occasionally
- Base-2 (binary) - most important! (we've been discussing 2 things!)
- Base-16 (hexadecimal) - nice grouping of bits

Binary

2 digits: 0, 1

Try to turn 1100101_2 into base-10: $= 1 \times 64 + 1 \times 32 + 1 \times 4 + 1 \times 1$

2^6 2^5 2^4 2^3 2^2 2^1 2^0

= = = = = = =

64 32 16 8 4 2 1

101_{10}

Binary

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048

2^0 2^1 2^2 2^3 2^4 2^5 2^6 2^7 2^8 2^9 2^{10} 2^{11}

Any downsides to binary?

Turn 2130_{10} into base-2:

hint: find largest power of 2 and subtract

what →

borrow →

$12 = 3 - 1 + 10$

$$
\begin{array}{r l}
2130 & \\
-\,2048 & = 2^{11} \\
\hline
0082 & \\
64 & = 2^{6} \\
\hline
18 & \\
16 & = 2^{4} \\
\hline
2 &
\end{array}
$$

1	0	0	0	0	1	0	1	0	0	1	0
2^{11}	2^{10}	2^{9}	2^{8}	2^{7}	2^{6}	2^{5}	2^{4}	2^{3}	2^{2}	2^{1}	2^{0}

Long Numbers

How do we deal with numbers too long to read?

37,625,034,653

Long Numbers

How do we deal with numbers too long to read?

- Group them by 3 (right to left)

Long Numbers

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- Group them by 3 (right to left)
- In decimal, use commas: ,
- Numbers between commas: 000 - 999 ← base 1000

Long Numbers

How do we deal with numbers too long to read?

- Group them by 3 (right to left)
- In decimal, use commas: ,
- Numbers between commas: 000 - 999
- Effectively base-1000

Long Numbers in Binary - Readability

- Typical to group by 3 or 4 bits
- No need for commas *Why?*

0, 1, 2, 3, 4, 5, 6, 7

$2^3 = 8$

\# positions

\# values

000
001
010
⋮
111

100001010010

Long Numbers in Binary - Readability

- Typical to group by 3 or 4 bits
- No need for commas *Why?*
- We can use a separate symbol per group
- How many do we need for groups of 3?

100001010010

Long Numbers in Binary - Readability

- Typical to group by 3 or 4 bits
- No need for commas *Why?*
- We can use a separate symbol per group
- How many do we need for groups of 3?
- Turn each group into decimal representation

$$\begin{array}{ccc} 2^2 & 2^1 & 2^0 \\ \shortparallel & \shortparallel & \shortparallel \\ 4 & 2 & 1 \end{array}$$

$$\overbrace{100}^{4}\,|\,\overbrace{001}^{1}\,|\,\overbrace{010}^{2}\,|\,\overbrace{010}^{2} = 4122_8$$

$$\begin{array}{ccc|ccc|ccc|ccc} 4 & 2 & 1 & 4 & 2 & 1 & 4 & 2 & 1 & 4 & 2 & 1 \end{array}$$

Long Numbers in Binary - Readability

- Typical to group by 3 or 4 bits
- No need for commas *Why?*
- We can use a separate symbol per group
- How many do we need for groups of 3?
- Turn each group into decimal representation
- Converts binary to **octal**

100001010010

Long Numbers in Binary - Readability

8 4 2 1

- Groups of 4 more common
- How many symbols do we need for groups of 4?

$2^4 = 16$

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8 5 2

852_{16}

Long Numbers in Binary - Readability

- Groups of 4 more common
- How many symbols do we need for groups of 4?
- Converts binary to **hexadecimal**
- Base-16 is very common in computing

100001010010

Hexadecimal

Need more than 10 digits. What next?

1110

e

0
1
⋮
9
a = 10
b = 11
c = 12
d = 13
e = 14
f = 15

Hexadecimal Exercise

Consider the following hexadecimal number:

852dab1e

1110

Is it even or odd?

Finally, Numbers!

Storing Integers

- Use binary representation of decimal numbers
- Usually have a limited number of bits (ex: 32, 64)
  - Depending on language
  - Depending on hardware