

# Floating Point Numbers (From the last class)

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## CS 2130: Computer Systems and Organization 1

**Xinyao Yi** Ph.D.  
Assistant Professor

## Floating Point Example

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$101.011_2$

## Floating Point Example

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What does the following encode?

1	001110	1010101
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What about 0?

## Floating Point Numbers

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Four cases:

- **Normalized:** What we have seen today

$$s \text{ } eeee \text{ } ffff = \pm 1.ffff \times 2^{eeee - \text{bias}}$$

- **Denormalized:** Exponent bits all 0

$$s \text{ } eeee \text{ } ffff = \pm 0.ffff \times 2^{1 - \text{bias}}$$

- **Infinity:** Exponent bits all 1, fraction bits all 0 (i.e.,  $\pm\infty$ )
- **Not a Number (NaN):** Exponent bits all 1, fraction bits not all 0

# More bits, circuits, adders

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## Announcements

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- Homework 1 due September 15

## Warm up!

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Can I make an  $n$ -input AND from 2-input AND gates?

What about XOR gates?



## Operations

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So far, we have discussed:

- Addition:  $x + y$ 
  - Can get multiplication
- Subtraction:  $x - y$ 
  - Can get division, but more difficult
- Unary minus (negative):  $-x$ 
  - Flip the bits and add 1

## Operations (on Integers)

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Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not:  $\sim x$  - flips all bits (unary)
- Bitwise and:  $x \& y$  - set bit to 1 if  $x, y$  have 1 in same bit
- Bitwise or:  $x | y$  - set bit to 1 if either  $x$  or  $y$  have 1
- Bitwise xor:  $x \wedge y$  - set bit to 1 if  $x, y$  bit differs

## Operations (on Integers)

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Logical not:  $!x$

- $!0 = 1$  and  $!x = 0, \forall x \neq 0$
- Useful in C, no booleans
- Some languages name this one differently

## Operations (on Integers)

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Left shift:  $x \ll y$  - move bits to the left

- Effectively multiply by powers of 2

Right shift:  $x \gg y$  - move bits to the right

- Effectively divide by powers of 2
- Signed (extend sign bit) vs unsigned (extend 0)

## Floating Point Numbers

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## Our Story So Far

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- Transistors
- Information modeled by voltage through wires (1 vs 0)
- Gates:  $\&$   $|$   $\sim$   $\wedge$
- Multi-bit values: representing integers
  - Signed and unsigned
  - Bitwise operators on bit vectors
- Floating point

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## How to do the work of multi-bit?

## Multi-bit Mux

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Our first multi-bit example: mux



## Adder

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Add 2 1-bit numbers:  $a$ ,  $b$

## Adder

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Can we use this in parallel to add multi-bit numbers?

## Adder

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Can we use this in parallel to add multi-bit numbers?

What is missing?

Consider:

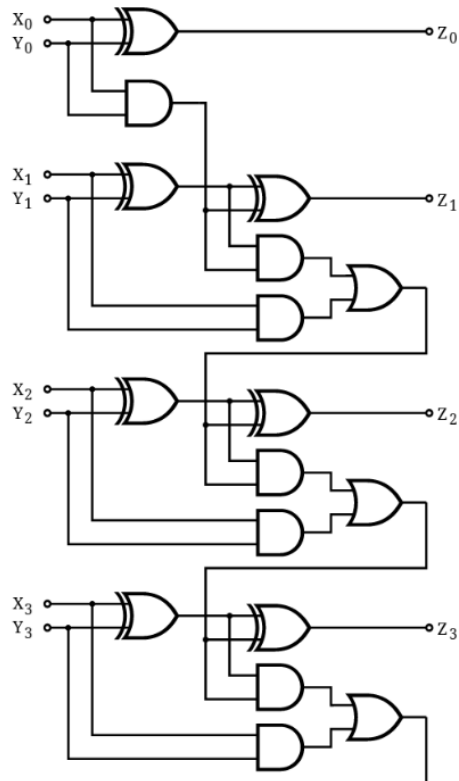
$$\begin{array}{r} 11 \\ +01 \\ \hline \end{array}$$

## 3-input Adder

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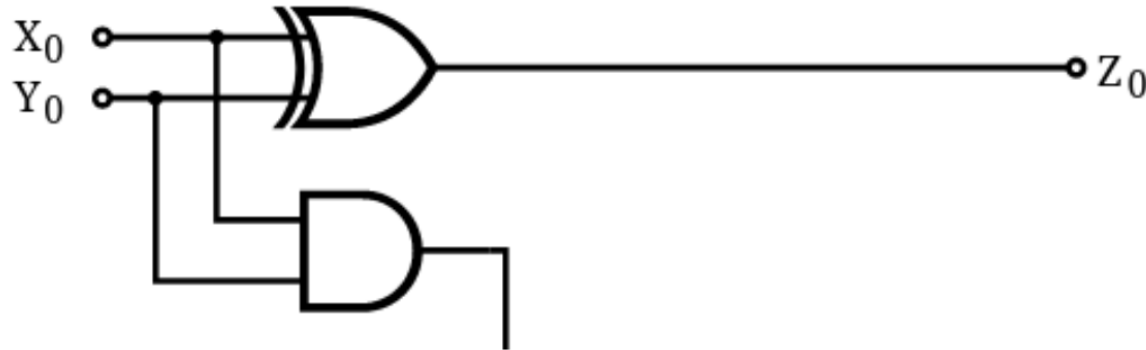
Add 3 1-bit numbers:  $a$ ,  $b$ ,  $c$

## Ripple-Carry Adder

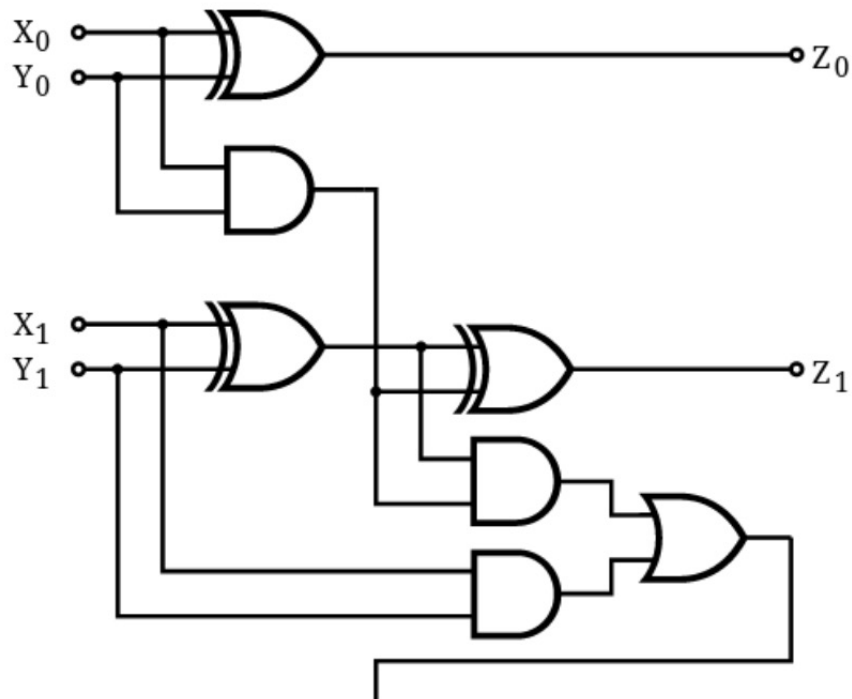


## Ripple-Carry Adder: Lowest-order Bit

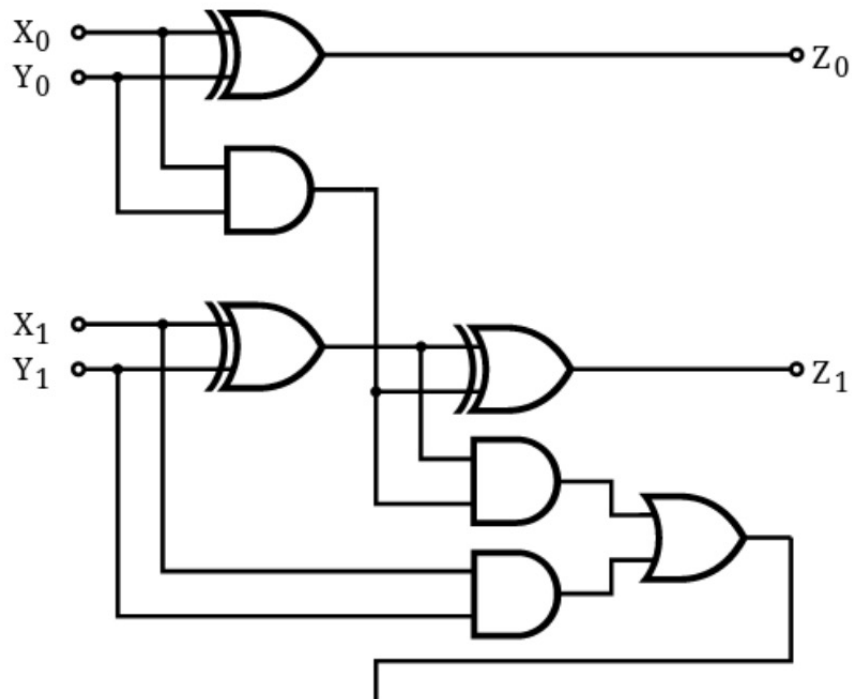
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## Ripple-Carry Adder: In General

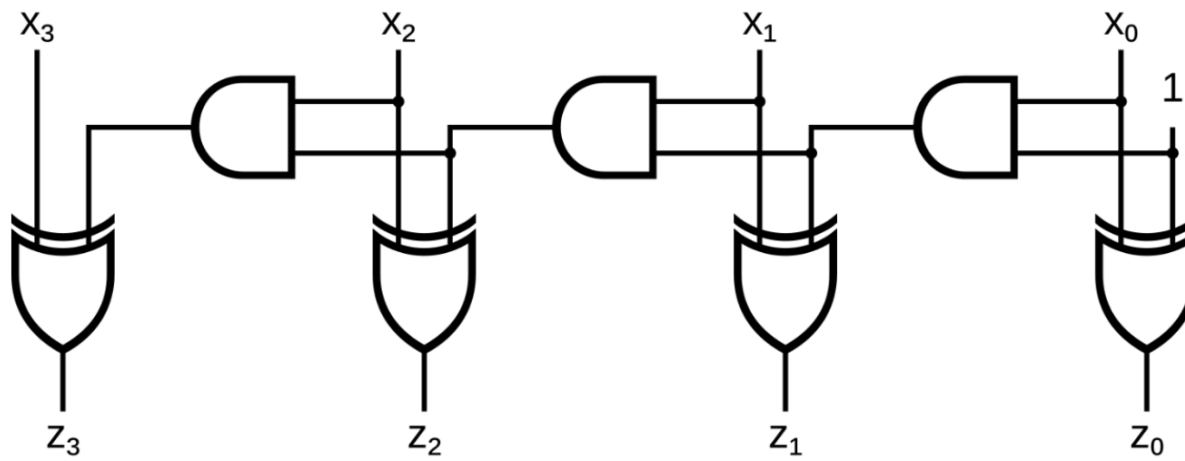


## Ripple-Carry Adder: In General

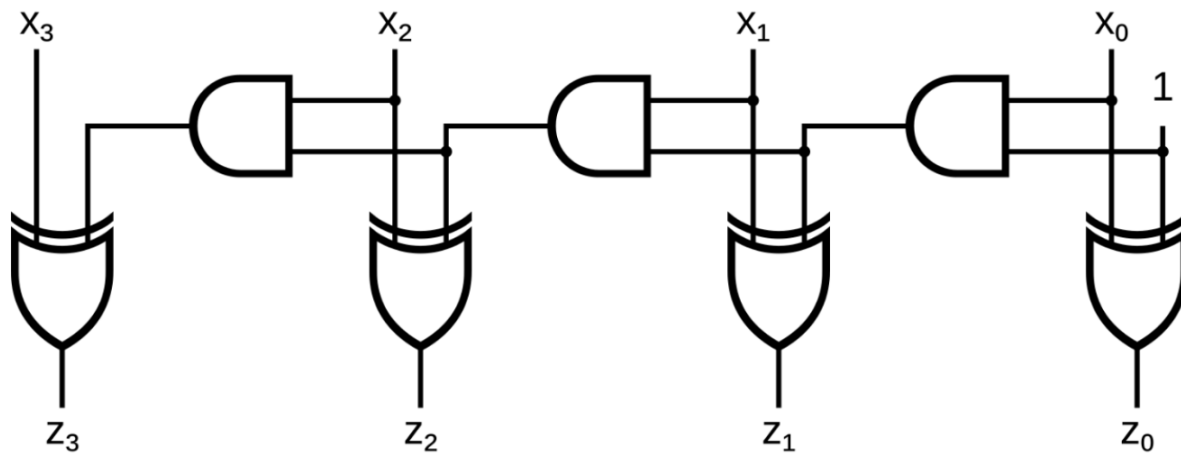




## What does this circuit do?



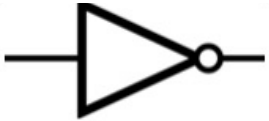
## Increment Circuit



## Gate Delay

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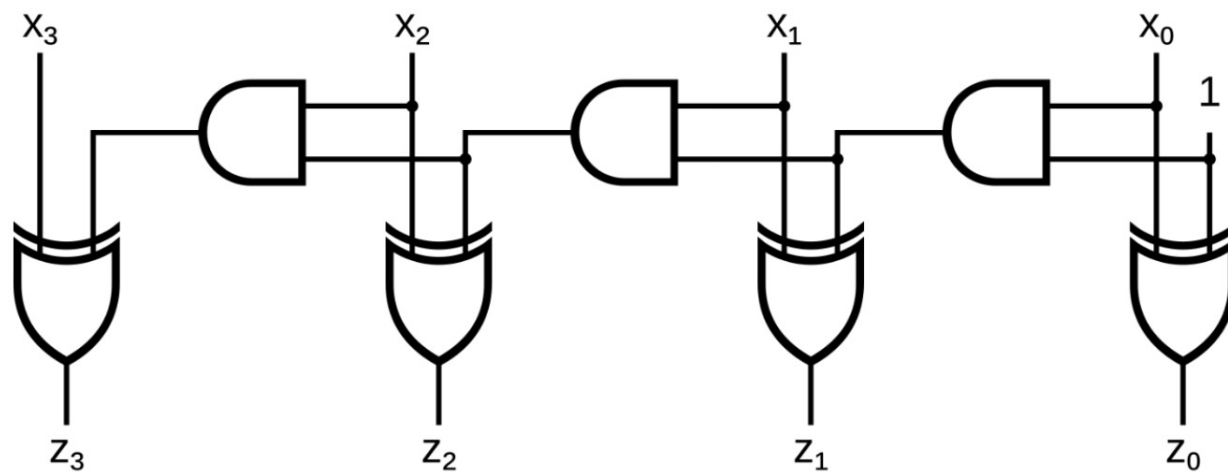
What happens when I change my input?



## Building a Counter

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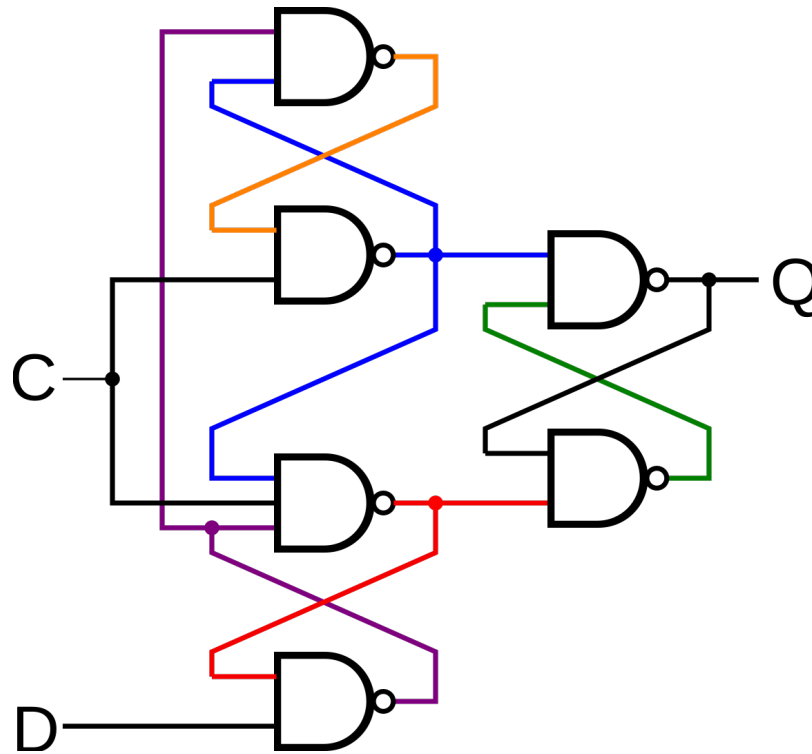
## Building a Counter



## **Building a Counter - Waiting**

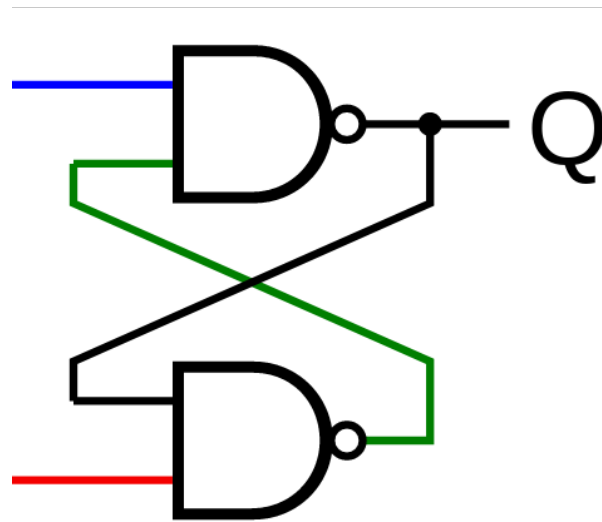
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## 1-bit Register Circuit



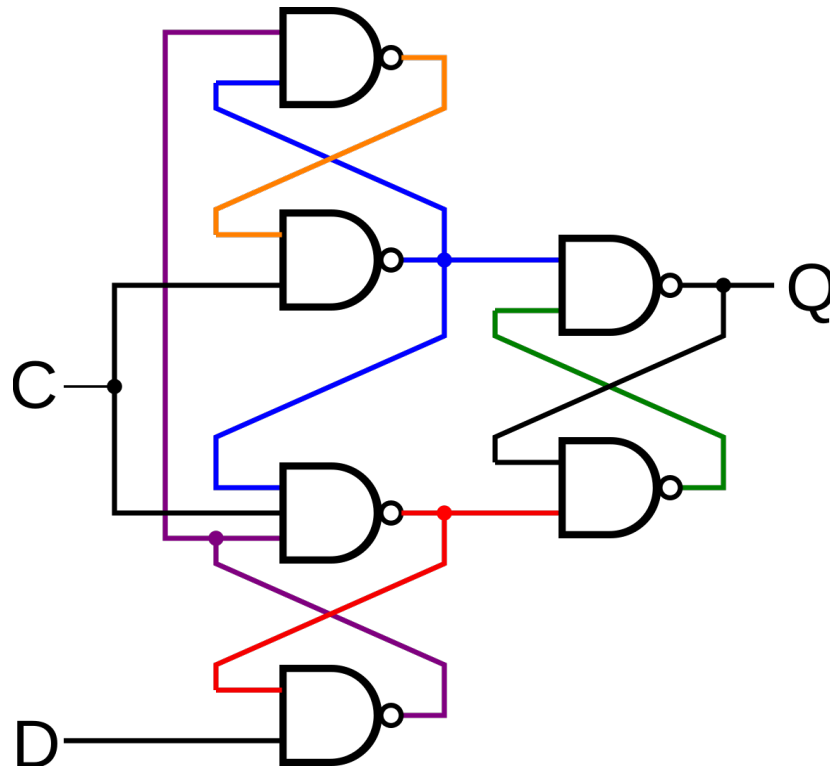
## 1-bit Register Circuit

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## 1-bit Register Circuit



## Building a Counter

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Any Questions?