Binary Arithmetic & Bitwise

CS 2130: Computer Systems and Organization 1

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Two's Complement

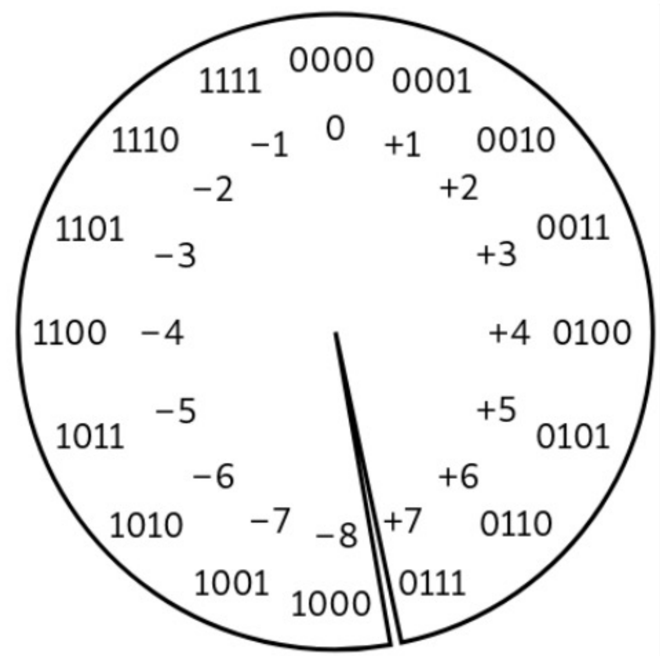
The scheme is called Two's Complement

Why do we need Two's Complement?

- We want the computer to represent both positive and negative numbers.
- And we want addition and subtraction to use the *same* hardware (just one adder), instead of building a separate "subtractor."

How does it work?

- The **leftmost bit (MSB)** is treated as negative.
  - In normal binary: the leftmost bit is +128 (for 8-bit).
  - In two's complement: the leftmost bit is −128.
- That's why $10000000_2 = -128$ instead of +128.



Values of Two's Complement Numbers

Consider the following 8-bit binary number in Two's Complement:

11010011

What is its value in decimal?

Values of Two's Complement Numbers

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What is its value in decimal?

1. Flip all bits
2. Add 1

① Flip all bits:

11010011 → 00101100

② Add 1: 00101100
+ 1
00101101

③ what is 00101101 in decimal?

00101101 = 32+8+4+1 = 45

④. So the value of this negtive number is −45.

Values of Two's Complement Numbers

Why "invert the bits and add 1"?

- Because in 8 bits, we have 256 total values (0–255).
- A negative number is stored as 256 − (its absolute value).
- The "invert + 1" trick is just a fast way to compute that.

Two's Complement

two's complement definition:

$-a = \sim a + 1$

$0 = \sim a + 1 + a$

$-1 = \sim a + a$

Values of Two's Complement Numbers

Consider the following decimal number:

-117

What is its value in 8-bit binary binary?

①. positive 117 in binary. 01110101

②. invert the bits: 10001010

③. +1 : 10001010 +1 = 10001011

Operations

So far, we have discussed:

- Addition: $x + y$
  - Can get multiplication
- Subtraction: $x - y$
  - Can get division, but more difficult
- Unary minus (negative): $-x$
  - Flip the bits and add 1

Operations (on Integers)

Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not: ~x - flips all bits (unary)
- Bitwise and: x & y - set bit to 1 if x, y have 1 in same bit
- Bitwise or: x | y - set bit to 1 if either x or y have 1
- Bitwise xor: x ^ y - set bit to 1 if x, y bit differs

Example: Bitwise AND

```
  11001010
& 01111100
----------
  01001000
```

Example: Bitwise OR

```
  11001010
| 01111100
----------
  11111110
```

Example: Bitwise XOR

```
   11001010
^  01111100
-----------
   10110110
```

Your Turn!

What is:

0x1a ^ 0x72

①. to binary:

1 a → 0001 1010

7 2 → 0111 0010

```
  0 0 0 1 1 0 1 0
^ 0 1 1 1 0 0 1 0
-----------------
  0 1 1 0 1 0 0 0
  \_____/ \_____/
     6       8
```

⇒ 0x68

Operations (on Integers)

Logical not: $!x$

- $!0 = 1$ and $!x = 0, \forall x \neq 0$
- Useful in C, no booleans
- Some languages name this one differently

Operations (on Integers)

Left shift: x << y - move bits to the left

- Effectively multiply by powers of 2

Right shift: x >> y - move bits to the right

- Effectively divide by powers of 2
- Signed (extend sign bit) vs unsigned (extend 0)

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Left Bit-shift Example

01011010 << 2

01011010 00

(01011010 × 2^2)

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Right Bit-shift Example

01011010 >> 3

00001011

(01011010/2^3)

Bit-shift

Computing bit-shift effectively multiplies/divides by powers of 2

Consider decimal:

$$2130 <<_{10} 2 = 213000 = 2130 \times 100$$

$$2130 >>_{10} 1 = 213 = 2130 / 10$$

Right Bit-shift Example 2

11001010 >> 1

if this is a signed number:

11100101

if this is an unsigned number:

01100101

Right Bit-shift Example 2

For signed integers, extend the sign bit (1)

- Keeps negative value (if applicable)
- Approximates divide by powers of 2

11001010 >> 1

Bit fiddling example

Any Questions?