Binary Arithmetic

CS 2130: Computer Systems and Organization 1

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Announcement

- My Office Hours (Rice 310 Or Zoom: https://virginia.zoom.us/j/3627787726)
  - Monday: 1 PM - 2 PM
  - Wednesday: 1 PM - 3 PM
- TA Office Hours starting today
- Homework 1 available Friday, due September 15, 2025

Binary

2 digits: 0, 1

Try to turn 1100101_2 into base-10

Binary

Any downsides to binary?

Turn 2130_{10} into base-2:

hint: find largest power of 2 and subtract

Long Numbers

How do we deal with numbers too long to read?

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- Group them by 3 (right to left)

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- Group them by 3 (right to left)
- In decimal, use commas: ,
- Numbers between commas: 000 – 999
- Effectively base-1000

Long Numbers in Binary - Readability

- Typical to group by 3 or 4 bits
- No need for commas *Why?*

100001010010

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- We can use a separate symbol per group
- How many do we need for groups of 3?
- Turn each group into decimal representation
- Converts binary to **octal**

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Long Numbers in Binary - Readability

- Groups of 4 more common
- How many symbols do we need for groups of 4?

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Long Numbers in Binary - Readability

- Groups of 4 more common
- How many symbols do we need for groups of 4?
- Converts binary to **hexadecimal**
- Base-16 is very common in computing

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Hexadecimal

Need more than 10 digits. What next?

1110

Hexadecimal Exercise

Consider the following hexadecimal number:

852dab1e

Is it even or odd?

Using Different Bases in Code

	Old Languages	New Languages
binary		
octal		
decimal		
hexadecimal		

Binary Addition

01101011 + 01100101

11101011 + 11100101

Binary Subtraction

01111011 - 01100101

Finally, Numbers!

Storing Integers

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- Usually have a limited number of bits (ex: 32, 64)
  - Depending on language
  - Depending on hardware

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- Use binary representation of decimal numbers
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- Is there something missing?

Negative Integers

Representing negative integers

- Can we use the minus sign?

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Representing negative integers

- Can we use the minus sign?
- In binary we only have 2 symbols, must do something else!

Two's Complement

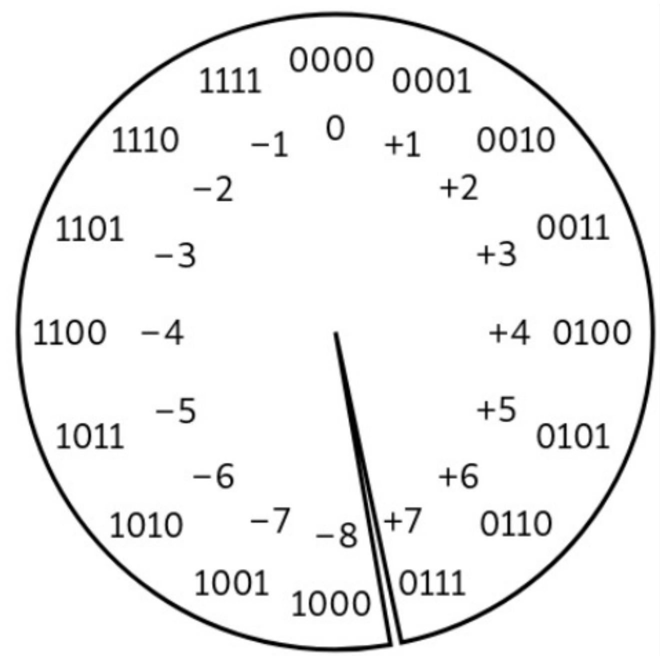
The scheme is called Two's Complement

Why do we need Two's Complement?

- We want the computer to represent both positive and negative numbers.
- And we want addition and subtraction to use the *same* hardware (just one adder), instead of building a separate "subtractor."

How does it work?

- The **leftmost bit (MSB)** is treated as negative.
  - In normal binary: the leftmost bit is +128 (for 8-bit).
  - In two's complement: the leftmost bit is −128.
- That's why $10000000_2 = -128$ instead of +128.



Values of Two's Complement Numbers

Consider the following 8-bit binary number in Two's Complement:

11010011

What is its value in decimal?

Values of Two's Complement Numbers

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What is its value in decimal?

1. Flip all bits
2. Add 1

Values of Two's Complement Numbers

Why "invert the bits and add 1"?

- Because in 8 bits, we have 256 total values (0–255).
- A negative number is stored as 256 − (its absolute value).
- The "invert + 1" trick is just a fast way to compute that.

Values of Two's Complement Numbers

Consider the following decimal number:

-117

What is its value in 8-bit binary binary?

Operations

So far, we have discussed:

- Addition: $x + y$
  - Can get multiplication
- Subtraction: $x - y$
  - Can get division, but more difficult
- Unary minus (negative): $-x$
  - Flip the bits and add 1

Operations (on Integers)

Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not: ~x - flips all bits (unary)
- Bitwise and: x & y - set bit to 1 if x, y have 1 in same bit
- Bitwise or: x | y - set bit to 1 if either x or y have 1
- Bitwise xor: x ^ y - set bit to 1 if x, y bit differs

Example: Bitwise AND

```
   11001010
&  01111100
-----------
```

Example: Bitwise OR

```
  11001010
| 01111100
----------
```

Example: Bitwise XOR

```
    11001010
^   01111100
------------
```

Your Turn!

What is:

0x1a ^ 0x72

Any Questions?