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Mux, Binary Arithmetic

CS 2130: Computer Systems and Organization 1

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Announcement

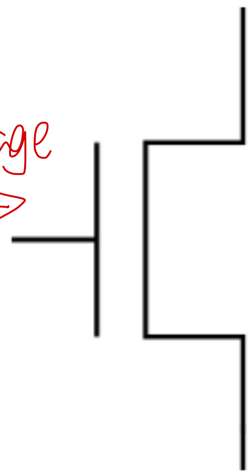
- **Lab grading clarification:** If you do not attend labs, the maximum score you can receive is **50%**, not 90%.
- **Slides updates:**
  - The annotated slides have been published on the course webpage.
  - Please note that in the future, I may occasionally make small updates on the slides after class—for example, adding more detailed explanations if many students found certain concepts difficult. I will let you know about any such changes at the beginning of the next class.

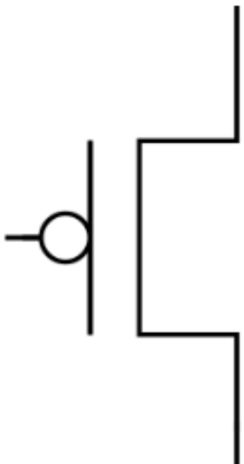
From Last Class…

Transistors

voltage.

Voltage





n-type transistor: Push to close → wire closed

- If we apply voltage, it switches **on**—current flows.
- If there's no voltage, it stays **off**.

p-type transistor: Push to open

- If we apply voltage, it actually switches **off**—it blocks the current.
- If there's no voltage, it stays **on**.

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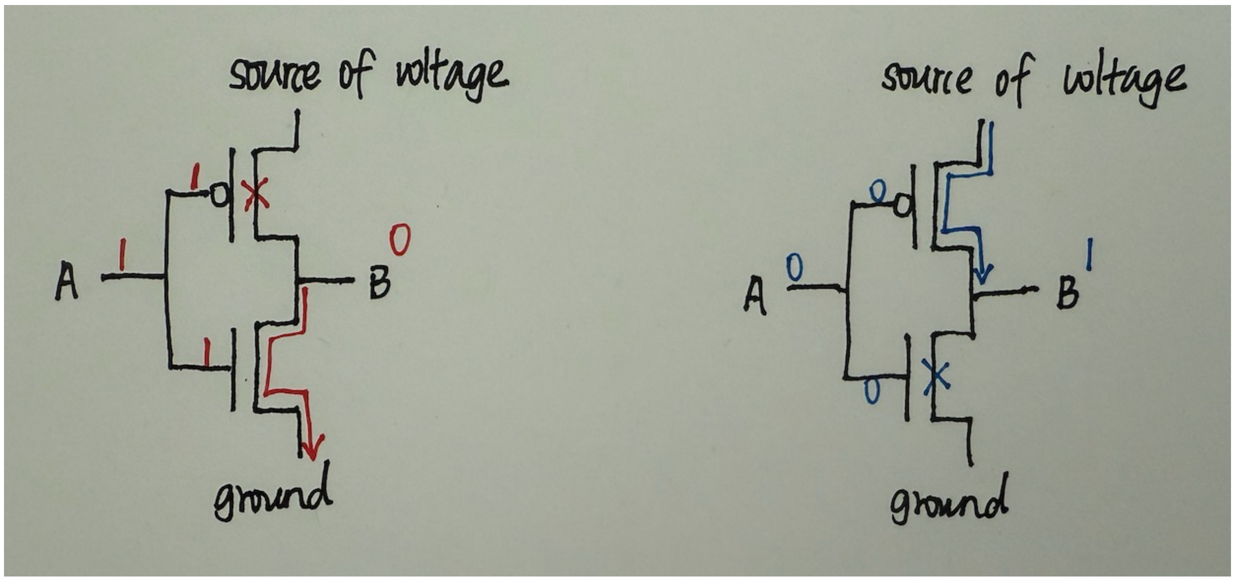
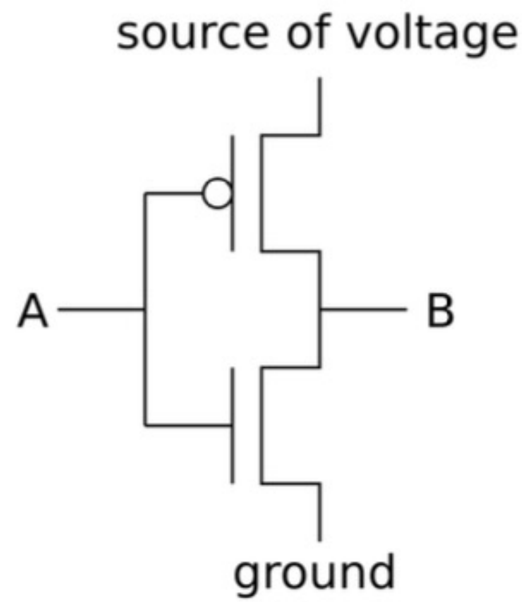
Circuit Diagram



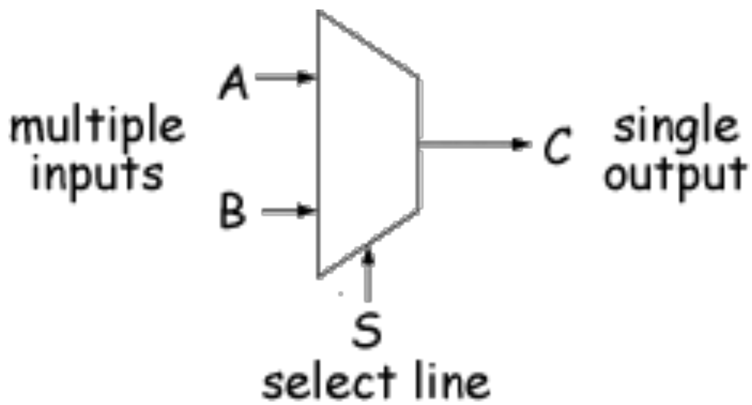


Multiplexer (mux)

x=a ? b : c

A multiplexer (mux) is commonly drawn as a trapezoid in circuit diagrams.

```
if(x=a){
    b
}
else {c }
```




```
if (S) {
    A
}
else {
    B
}
```

Multiplexer (mux)

S ? A : B

S	A	B	C
0			B
1			A

if S ⇒ B

if S ⇒ A

S	A	B	S
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Multiplexer (mux)

S	A	B	S
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

True:

1. !S & !A & B
2. !S & A & B
3. S & A & !B
4. S & A & B

Combine:

(!S& !A &B) | (!S&A&B) |

(S& A& !B) | (S&A&B)

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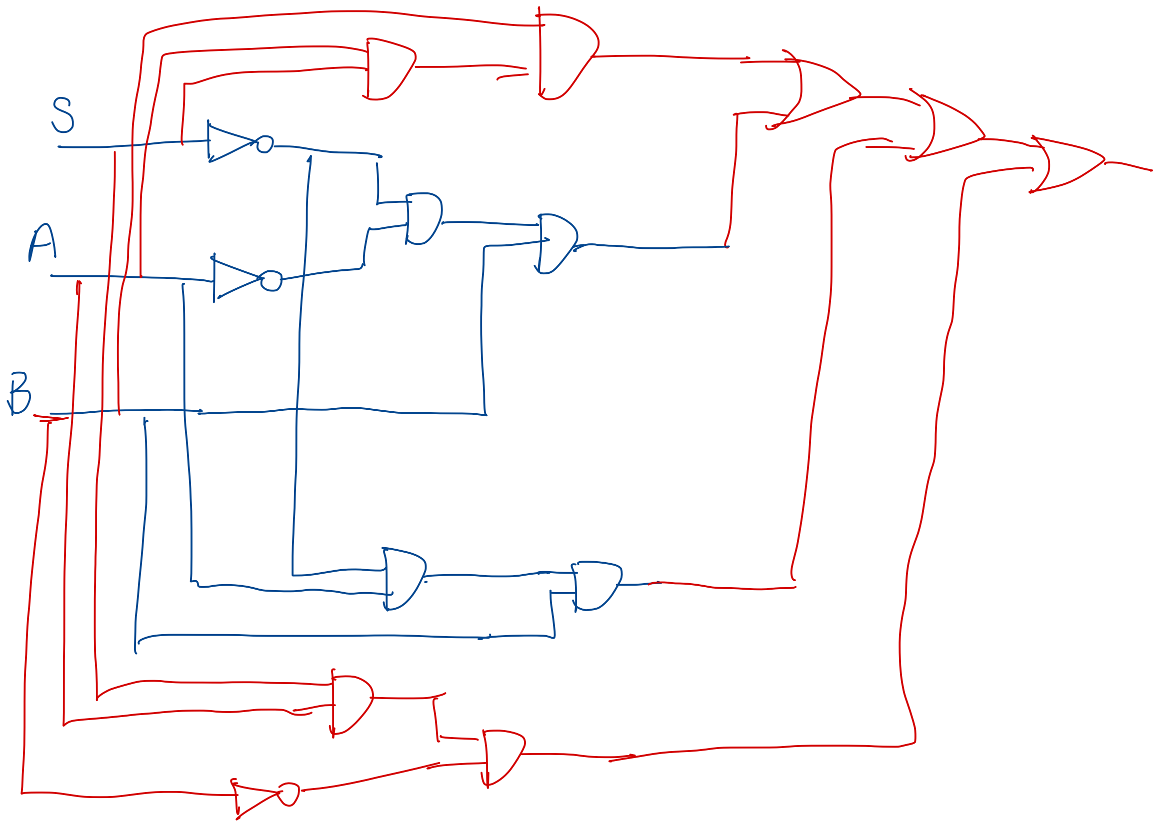
Multiplexer (mux)

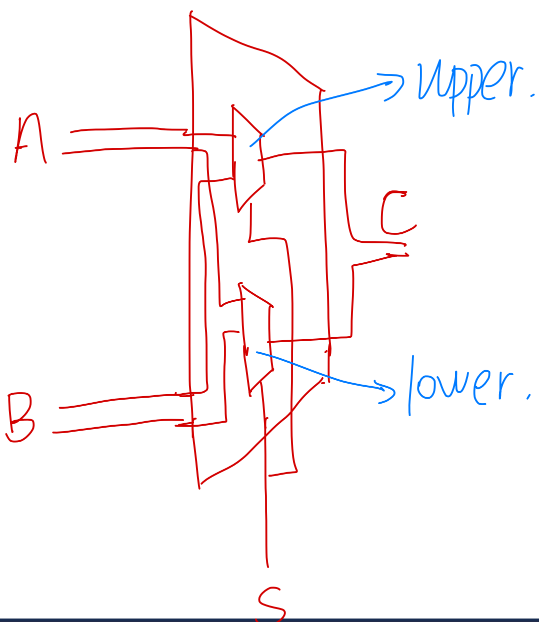
(!S& !A &B) | (!S&A&B) |

(S& A& !B) | (S&A&B)



2-bit Multiplexer (mux)

2-bit values instead of 1-bit values



In parallel.

Multi-bit Values

So far, only talking about 2 things: 0 and 1

Next:
Numbers, strings, objects, ...

Numbers

From our oldest cultures, how do we mark numbers?

- unary representation: make marks, one per ”thing”
    - Awkward for large numbers, ex: CS 2130?
    - Hard to tell how many marks there are
- Update: group them!
- Romans used new symbols:

V V

5 50 100 500 1000
V L C D M

(at least get this shorter

Numbers

Arabic numerals

- Positional numbering system

the position means something.

decimals:

2000 100 30 0

2 1 3 0

10^3 10^2 10^1 10^0

$2 \times 1000 + 1 \times 100 + 3 \times 10 + 0 \times 1 = 2130$

Numbers

Arabic numerals

- Positional numbering system
- The 10 is significant:
  - 10 symbols, using 10 as base of exponent

Numbers

Arabic numerals

- Positional numbering system
- The 10 is significant:
  - 10 symbols, using 10 as base of exponent
- The 10 is arbitrary
  - We can use other bases! π, 2130, 2, ... $\rightarrow$ binary.

Base-8 Example

Try to turn 134_8 into base-10:

$$\begin{array}{ccccc} & & 1 & 3 & 4 \\ \_ & \_ & \_ & \_ & \_ \\ 8^4 & 8^3 & 8^2 & 8^1 & 8^0 \\ & & 64 & 8 & 1 \end{array}$$

$$1 \times 64 + 3 \times 8 + 4 \times 1 = 92_{10}$$

Bases

We will discuss a few in this class

- Base-10 (decimal) - talking to humans
- Base-8 (octal) - shows up occasionally
- Base-2 (binary) - most important! (we've been discussing 2 things!)
- Base-16 (hexadecimal) - nice grouping of bits

Binary

2 digits: 0, 1

Try to turn 1100101_2 into base-10

1	1	0	0	1	0	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

→ Don't need to memorize, but familiar to ...

$$1 \times 64 + 1 \times 32 + \cancel{0 \times 16} + \cancel{0 \times 8} + 1 \times 4 + \cancel{0 \times 2} + 1 \times 1 =$$

$$64 + 32 + 4 + 1 = 101_{10}$$ (Only additions needed).

Binary

Any downsides to binary?

2^{11} 2^{10} 2^{9} 2^{8} 2^{7} 2^{6} 2^{5} 2^{4} 2^{3} 2^{2} 2^{1} 2^{0}

2048 1024 512 256 128 64 32 16 8 4 2 1

1 0 0 0 0 1 0 1 0 0 1 0

Turn 2130_{10} into base-2:

2130 - 2048 = 82   82 - 64 = 18   18 - 16 = 2

hint: find largest power of 2 and subtract

Long Numbers

How do we deal with numbers too long to read?

Long Numbers

How do we deal with numbers too long to read?

- Group them by 3 (right to left)