Boolean Algebra

CS 2130: Computer Systems and Organization 1

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Announcement

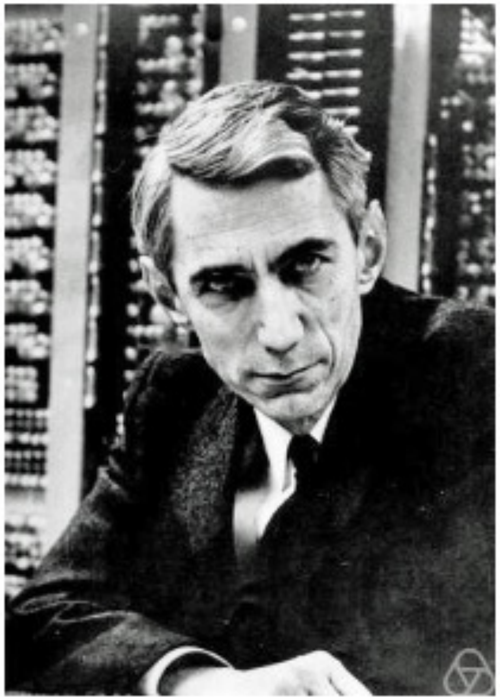
1. If you need to switch labs:
   - Form will be coming soon
   - Must be justified (i.e. class conflicts)
   - Very limited space to make swaps

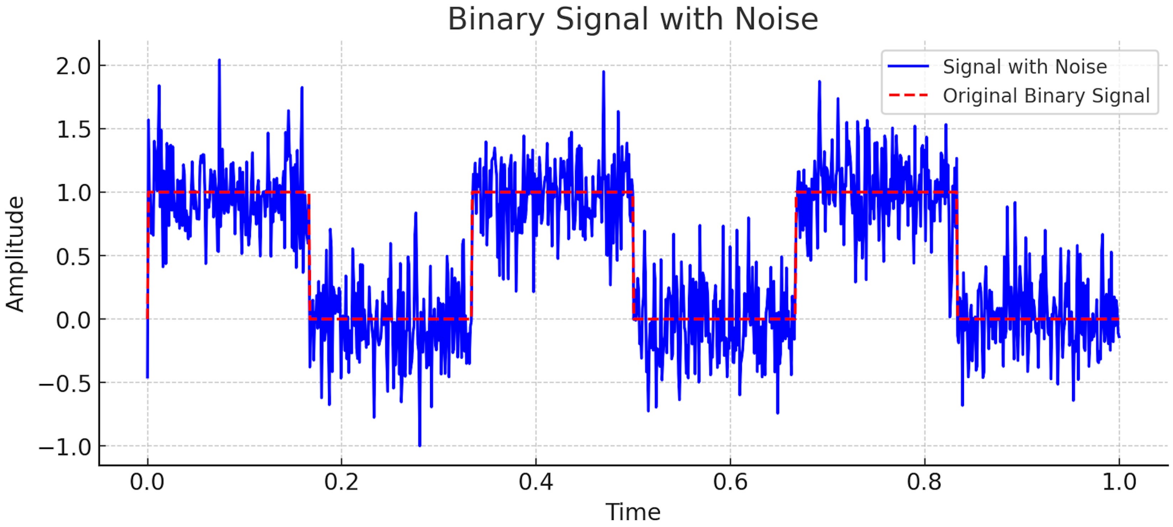
2. Quiz 1 opens tonight, due Sunday 11:59pm

3. For the Exams: The question types will be similar to those used in **Spring 2023**

Why only 0 and 1?

Claude Shannon





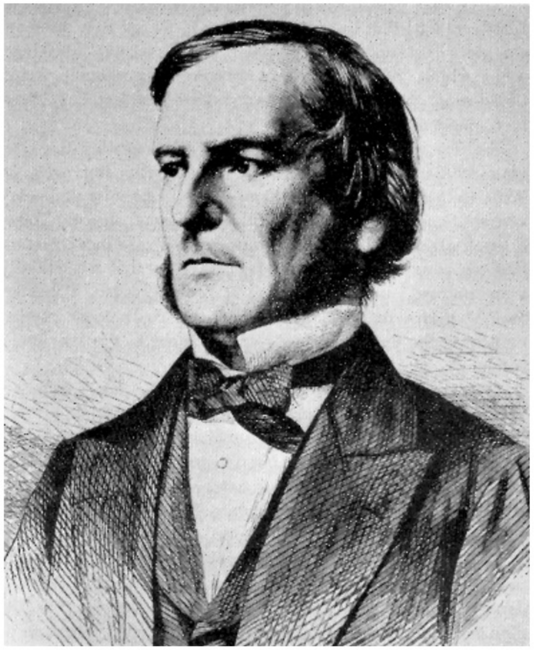
Vocabulary

- bit – either a 0 or 1

- binary - a system that has only two positions
- trinary - a system that has only three positions
- quadrinary - a system that has only four positions
- ...
- decinary - ...
- decimal - system that has ten positions

Boolean Algebra

George Boole



In Boolean Algebra, we live in a world with only two values:

- **True or False**
- **Yes or No**
- **1 or 0**

Boole showed that you could build an entire algebraic system using only these two values.

And that system uses three basic operations: **AND, OR, and NOT**.”

Putting Them Together

Overall idea:

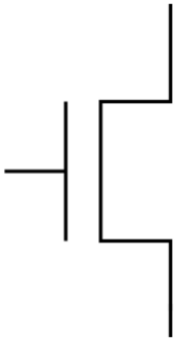
- Only need two things (Shannon)
- We can do math with two things (Boole)

Now we need a physical device that deals in two levels

Transistors

Electricity (conceptually) - involves flow of electrons or other charged carriers through a conductive material

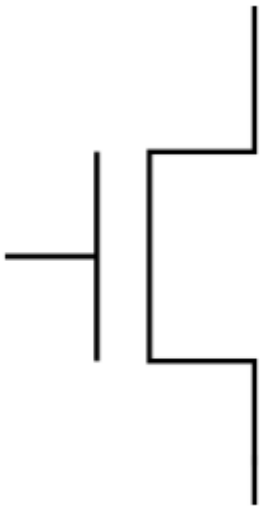
- current - rate of flow
- voltage - pressure of flow



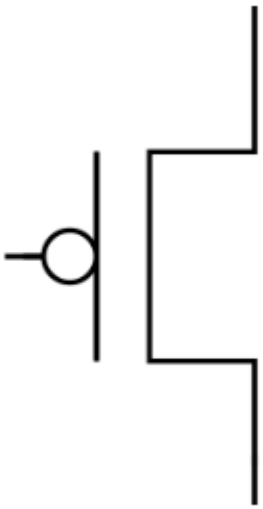
Transistors act like an electrically-triggered switch

- No voltage, no current
- Apply voltage to allow current to flow
- The amount of voltage needed to open the gate is boundary between 0 and 1
- Central technique for how we are going to build binary computers

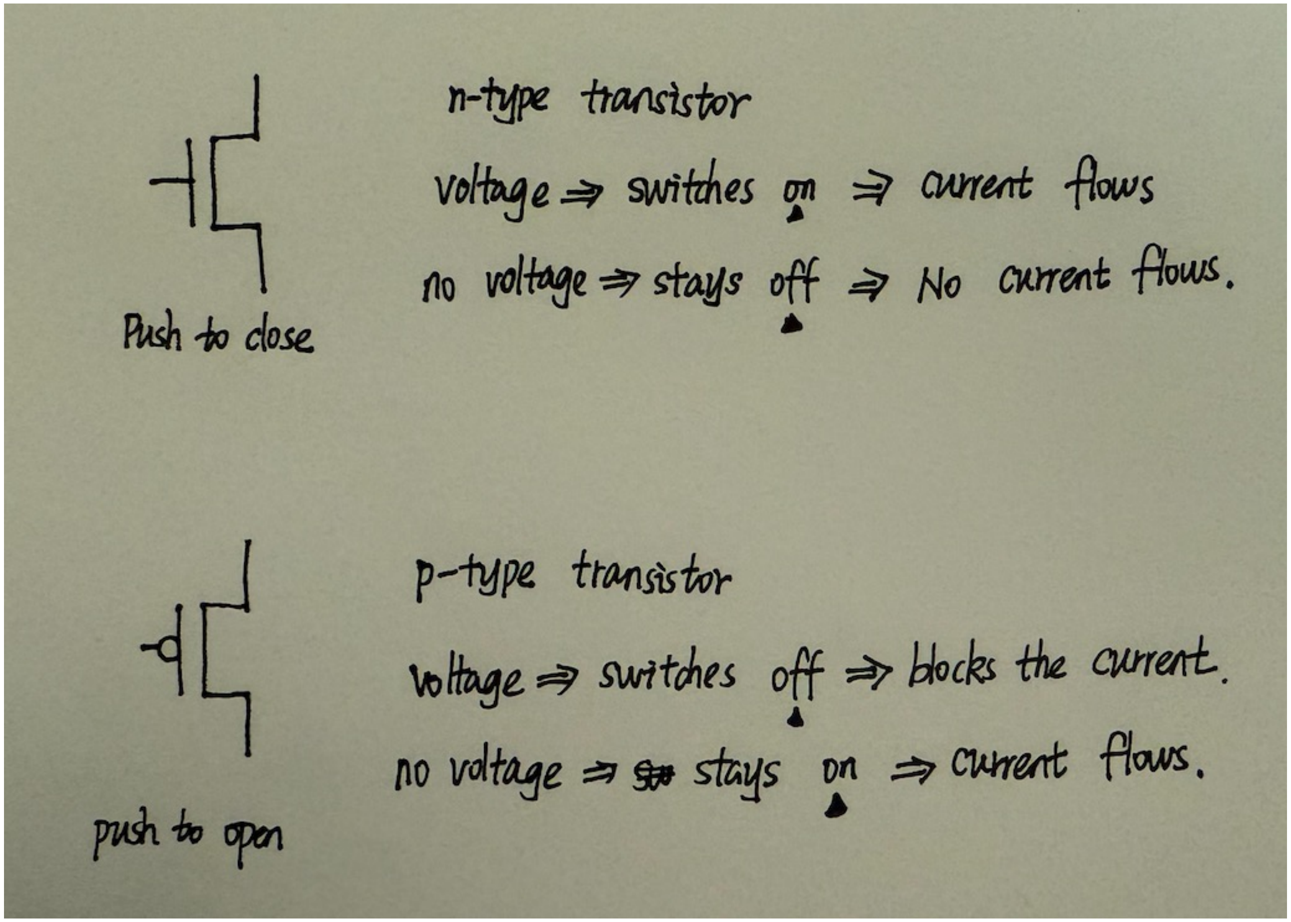
Transistors



Push to close



Push to open



n-type transistor

voltage ⇒ switches on ⇒ current flows

no voltage ⇒ stays off ⇒ No current flows.

Push to close

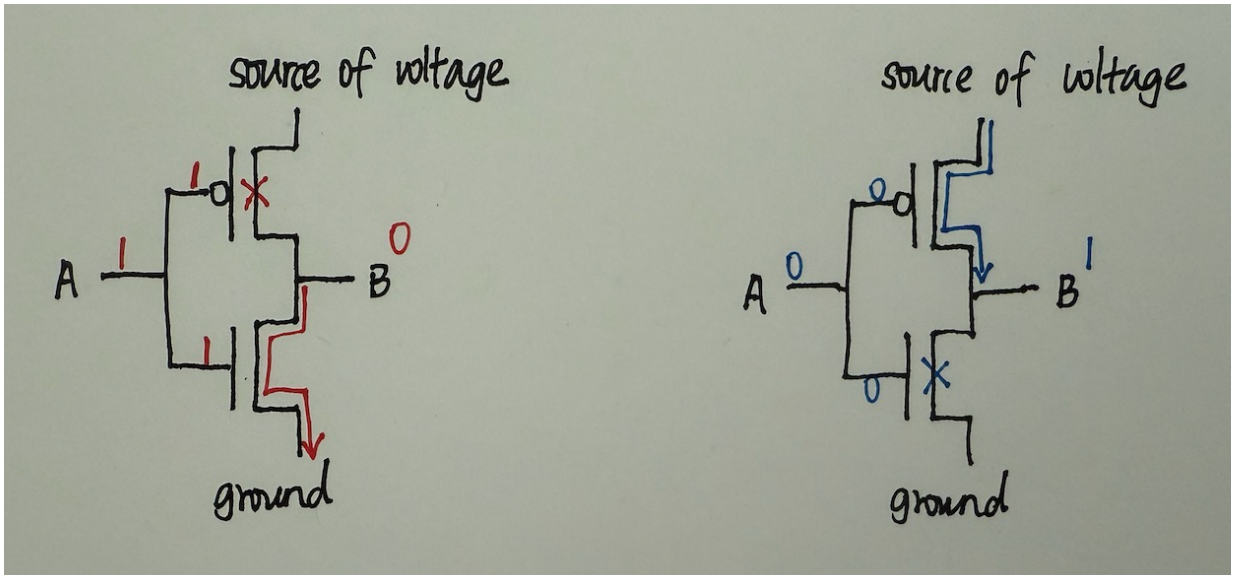
p-type transistor

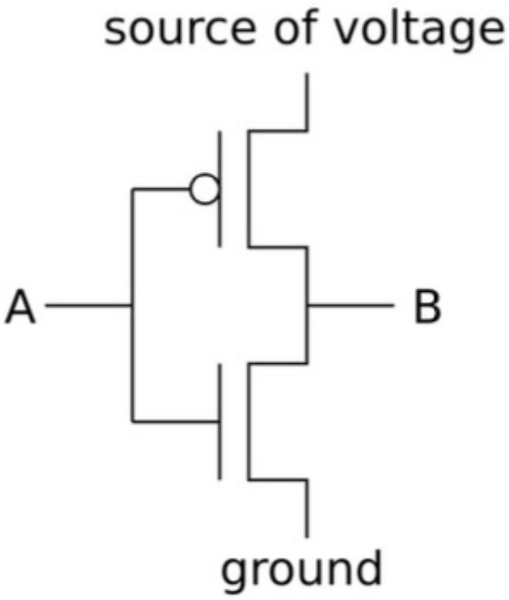
voltage ⇒ switches off ⇒ blocks the current.

no voltage ⇒ ~~st~~ stays on ⇒ current flows.

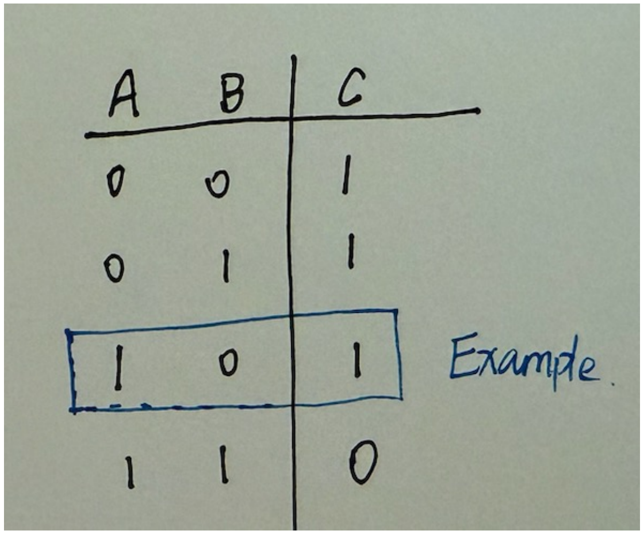
push to open

Circuit Diagram



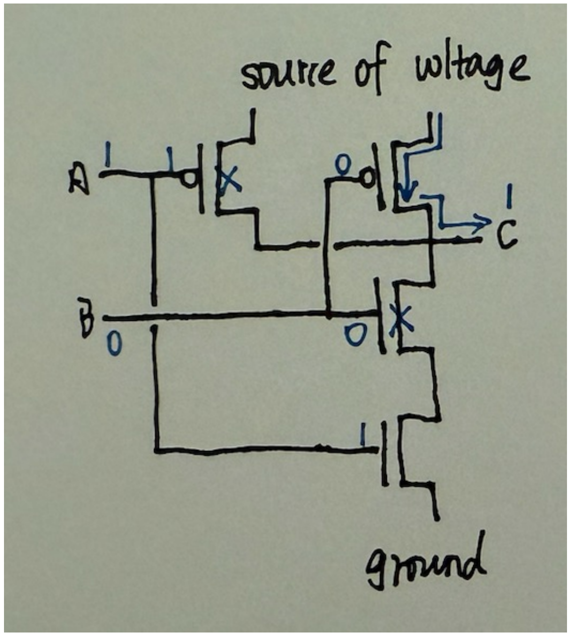


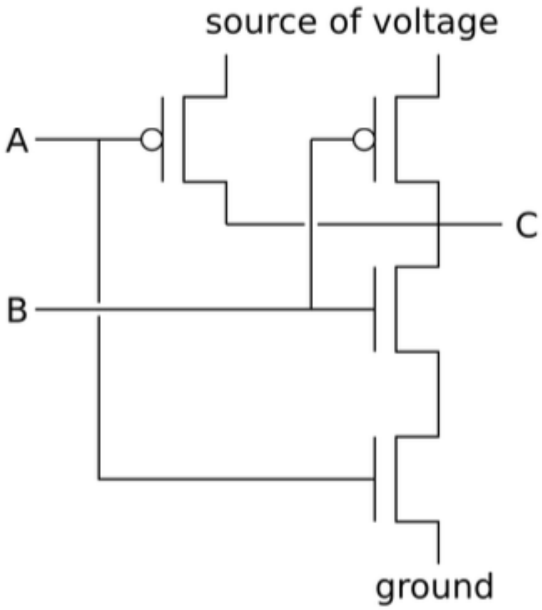
Circuit Diagram



A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

Example





Other Gates

Reading: https://uva-cs.github.io/cso1-f25/readings/bool.html

Building Up

Where we are now

- World with only 2 states: 0 and 1
- Re-developed Boolean logic: and, or, not

Gives us everything Boole talked about

- We can do a lot of interesting things!
- Next: build higher level ideas: the trinary operator

Trinary Operator

General idea

```
if ( … ) {
    …
} else {
    …
}
```

Trinary operator (expression if)

Python:

```
x=b if a else c
```

Java:

```
x=a?b:c
```

Multiplexer (mux)

How can we build a mux out of what we have learned so far?

x=a ? b : c

Can be built from and, or, and not

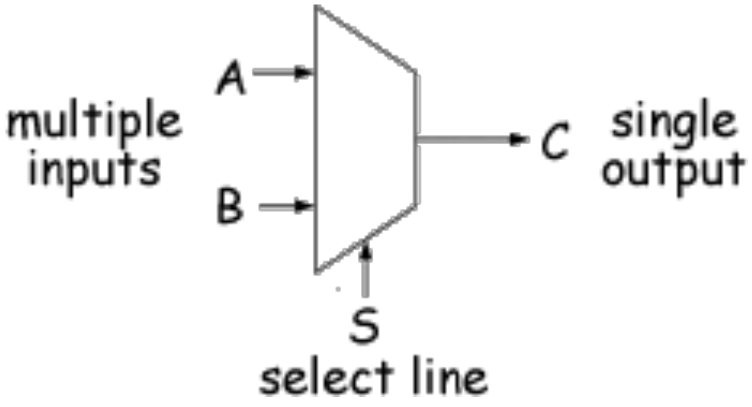
- Can be built using transistors
- Can physically put it in silicon!

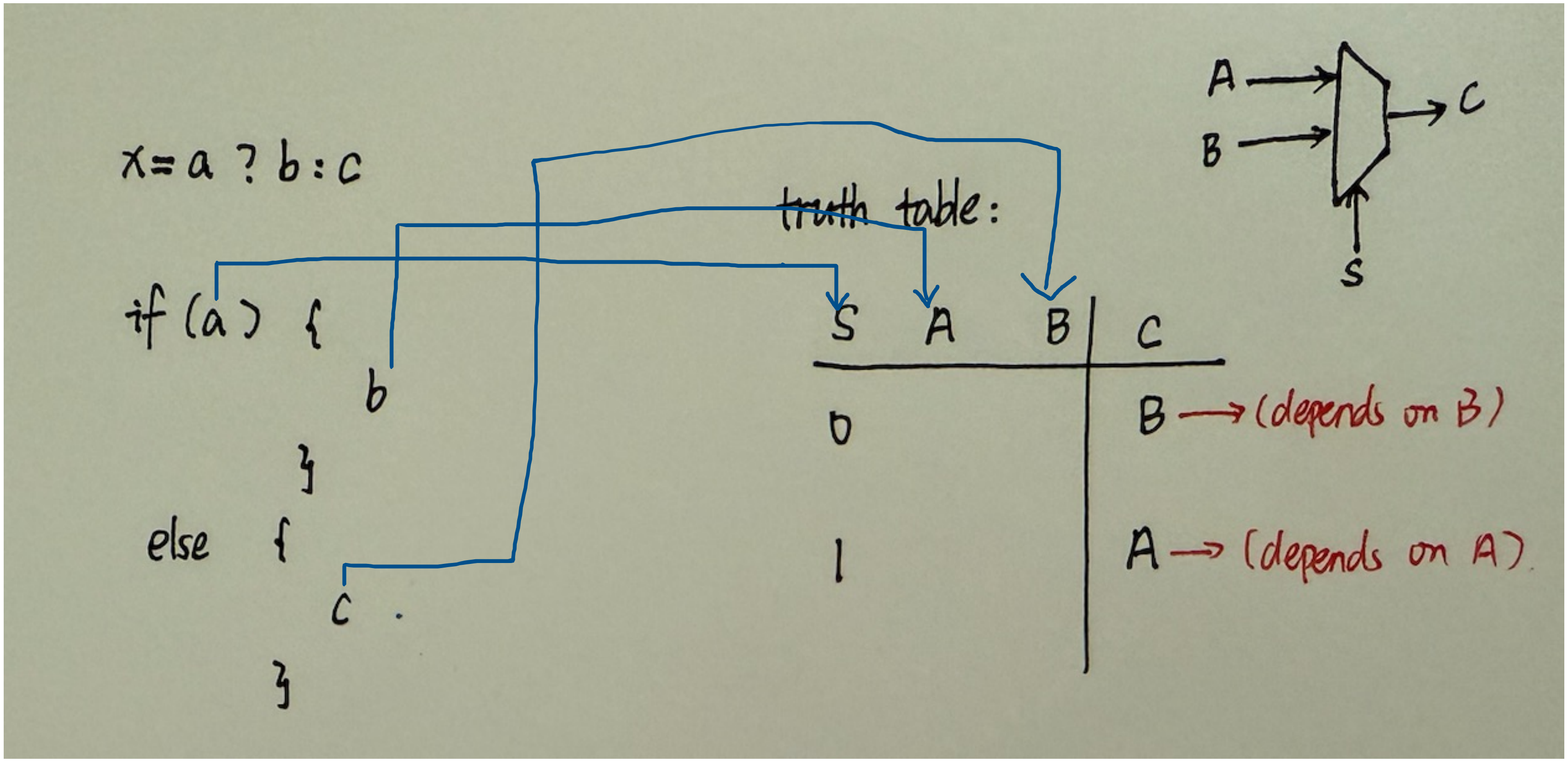
Mux will be the key when constructing a computer out of gates and circuits!

Multiplexer (mux)

x=a ? b : c

A multiplexer (mux) is commonly drawn as a trapezoid in circuit diagrams.



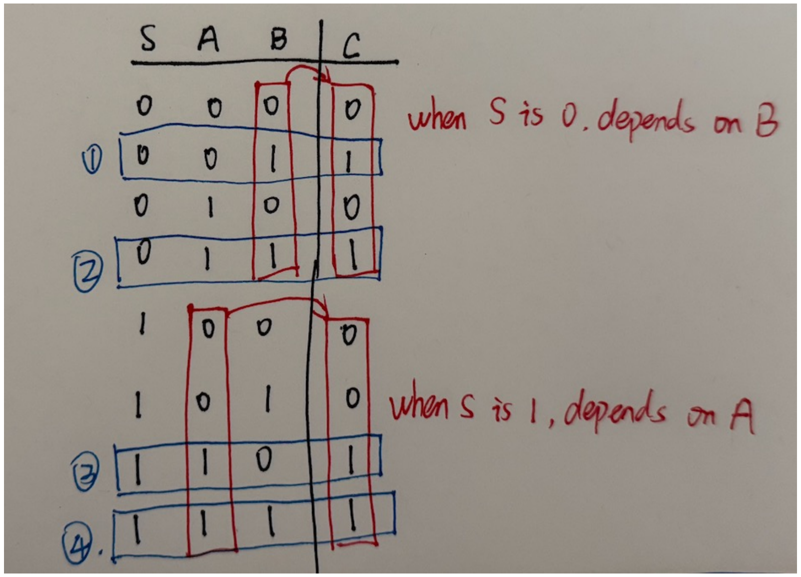


```
x= a ? b : c

if (a) {
        b
      }
 else {
        c .
      }
```

truth table:

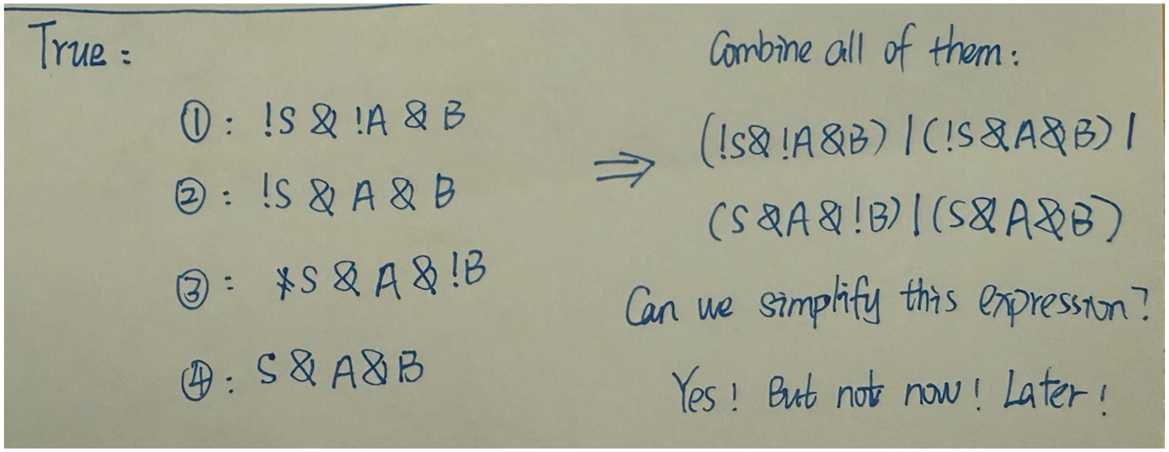
S	A	B	C
0			B → (depends on B)
1			A → (depends on A)



	S	A	B	C
	0	0	0	0
①	0	0	1	1
	0	1	0	0
②	0	1	1	1
	1	0	0	0
	1	0	1	0
③	1	1	0	1
④.	1	1	1	1

when S is 0, depends on B

when S is 1, depends on A



True :

① : !S & !A & B

② : !S & A & B

③ : ~~!~~S & A & !B

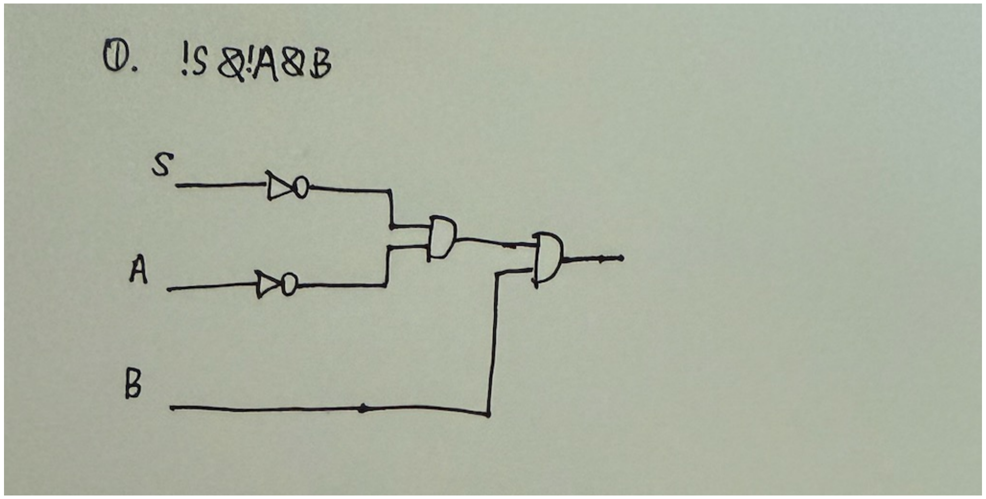
④ : S & A & B

Combine all of them:

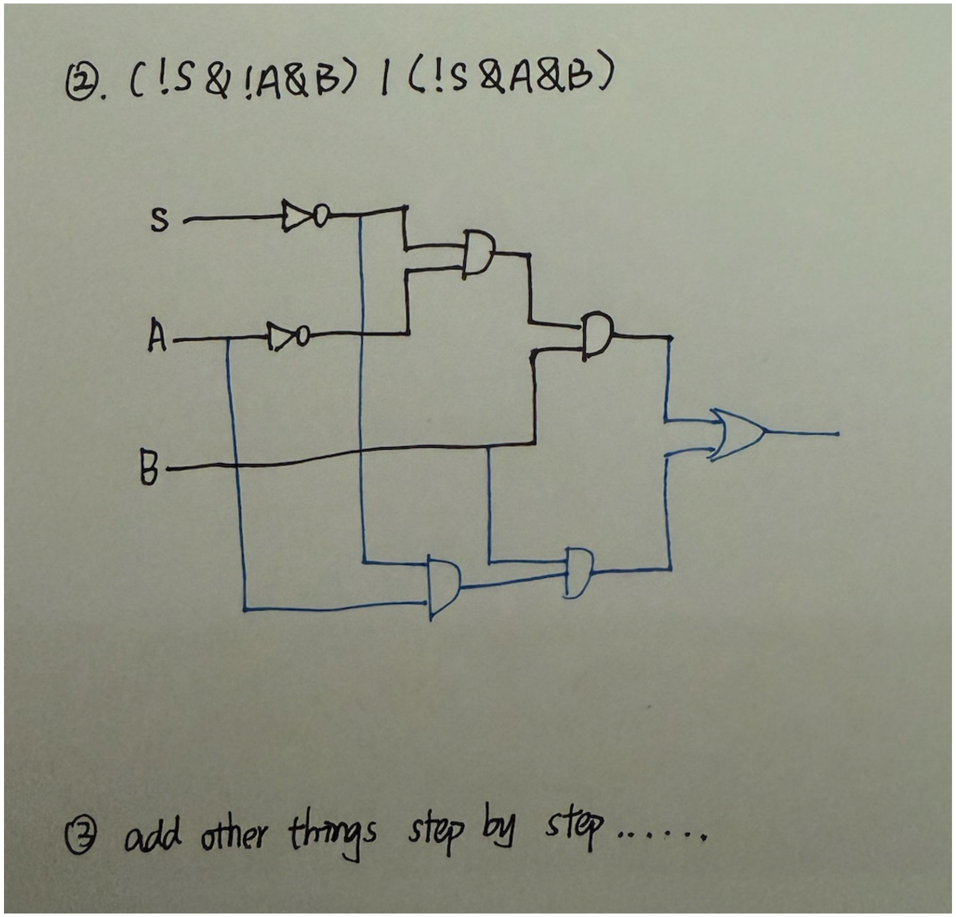
⇒ (!S& !A&B) | (!S&A&B) | (S&A&!B) | (S&A&B)

Can we simplify this expression?

Yes! But not now! Later!



①. !S &!A&B



②. (!S & !A&B) | (!S &A&B)

③ add other things step by step ......

2-bit Multiplexer (mux)

2-bit values instead of 1-bit values

Any Questions?