

# Using Different Bases in Code

How do we define numbers in our code (C, Java, Python, ...)?

	<i>C, Java?</i> Old Languages	<i>Python, ...</i> New Languages
binary	<i>No!</i>	<i>0b11010010</i>
octal	<i>02130</i>	<i>0o2130 <del>02130</del> 0o72</i>
decimal	<i>2130</i>	<i>2130</i>
hexadecimal	<i>0x86</i>	<i>0x86</i>

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# Bitwise Operations

CS 2130: Computer Systems and Organization 1  
September 5, 2025

# Announcements

- Quiz 1 opens this afternoon, due Sunday night
- Homework 1 due September 15
- SDAC is looking for a volunteer notetaker for our section
  - Please consider signing up and adding your notes
  - More details coming in the weekly email today

# Representing Negative Integers

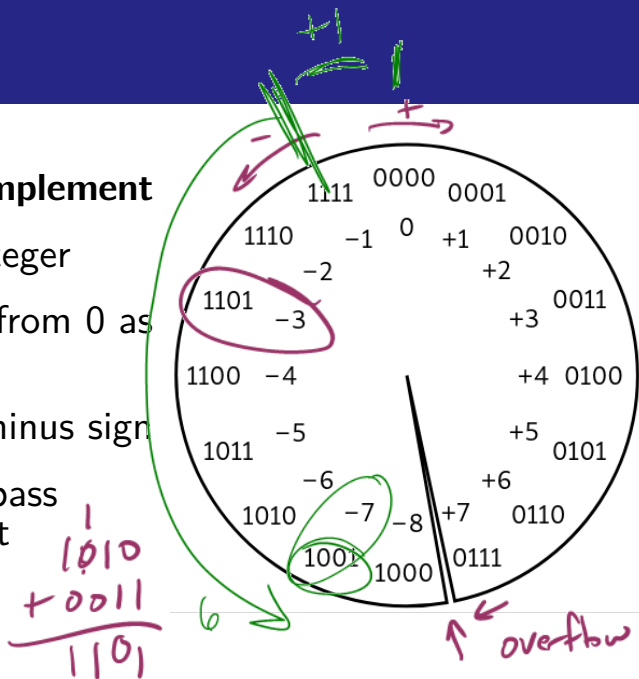
Computers store numbers in fixed number of wires

- Ex: consider 4-digit decimal numbers
- Throw away the last borrow:
  - $10000 - 0001 = 9999 == -1$
  - $9999 - 0001 = 9998 == -2$
  - Normal subtraction/addition still works
  - Ex:  $-2 + 3$
- This works the same in binary

# Two's Complement

This scheme is called **Two's Complement**

- More generically, a *signed* integer
- There is a break as far away from 0 as possible
- First bit acts vaguely like a minus sign
- Works as long as we do not pass number too large to represent




# Two's Complement

# Values of Two's Complement Numbers

Consider the following 8-bit binary number in Two's Complement:

11010011  $\leftarrow -45$

What is its value in decimal?


$$\begin{array}{r} 11111111 \leftarrow \text{carry bits} \\ 11010011 \\ + 00101101 \\ \hline 00000000 \end{array}$$

$$\begin{array}{r} 11111111 \leftarrow (-1) \\ -11010011 \\ \hline 00101100 \\ + 1 \\ \hline 00101101 = 45 \end{array}$$

32 16 8 4 2 1

# Values of Two's Complement Numbers

Consider the following 8-bit binary number in Two's Complement:

11010011

What is its value in decimal?

1. Flip all bits ~
2. Add 1



# Addition

$$\begin{array}{r}
 \begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \swarrow \text{Carry} \\
 01001010 = 74 \\
 + 01111100 = 124 \\
 \hline
 11000110 = \cancel{198} ? = 58
 \end{array}$$

$$\begin{array}{r}
 00111001 \\
 + \quad \quad \quad 1 \\
 \hline
 00111010 = 58
 \end{array}$$

# Subtraction

$$0111 \text{ --- } 1 = \frac{127}{\text{borrow}} \downarrow -128$$

$$\begin{array}{cccccccc} & 1 & 10 & 10 & 10 & 10 & 10 & 10 \\ & \rightarrow & - & - & - & - & - & - \end{array} \quad (2)$$

$$01001010 = 74$$

$$- 01111100 = 124$$

$$\underline{11001110} = -50$$

$$\begin{array}{cccccccc} 2^{256} & 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ \sim & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array} \leftarrow 49$$

$$+1 \quad \quad \quad 1 \leftarrow 1$$

$$\underline{00110010} \leftarrow 50$$

$$\begin{array}{r} 110 \\ 14 \\ -18 \\ \hline \end{array}$$

# Operations

So far, we have discussed:

- Addition:  $x + y$ 
  - Can get multiplication
- Subtraction:  $x - y$ 
  - Can get division, but more difficult
- Unary minus (negative):  $-x$ 
  - Flip the bits and add 1

# Operations (on Integers)

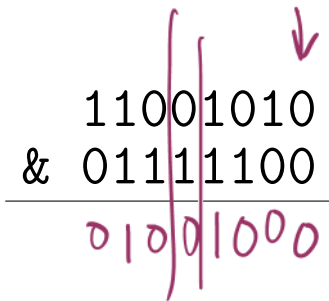
Bit vector: fixed-length sequence of bits (ex: bits in an integer)

- Manipulated by bitwise operations

Bitwise operations: operate over the bits in a bit vector

- Bitwise not:  $\sim x$  - flips all bits (unary)
- Bitwise and:  $x \ \& \ y$  - set bit to 1 if  $x, y$  have 1 in same bit
- Bitwise or:  $x \ | \ y$  - set bit to 1 if either  $x$  or  $y$  have 1
- Bitwise xor:  $x \ ^ \ y$  - set bit to 1 if  $x, y$  bit differs

# Example: Bitwise AND

$$\begin{array}{r} 11001010 \\ \& 01111100 \\ \hline 01001000 \end{array}$$


# Example: Bitwise OR

↓

	1	1	0	0	1	0	1	0
	0	1	1	1	1	1	0	0
<hr/>								
	1	1	1	1	1	1	1	0

# Example: Bitwise XOR

$$\begin{array}{r} 11001010 \\ \wedge 01111100 \\ \hline 10110110 \end{array}$$

# Your Turn!

What is:  $0x1a \wedge 0x72$

$$0x1a = \overset{1}{\underline{0}} \overset{a}{\underline{0}} \overset{a}{\underline{0}} \overset{a}{\underline{1}} \quad \overset{a}{\underline{1}} \overset{a}{\underline{0}} \overset{a}{\underline{1}} \overset{a}{\underline{0}}$$

$$0x72 = \underline{0} \underline{1} \underline{1} \underline{1} \quad \underline{0} \underline{0} \underline{1} \underline{0}$$

7                      2

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$$\underline{\underline{0110}} \quad \underline{\underline{1000}} = 0x68$$

4                      8



# Operations (on Integers)

- Logical not:  $!x$ 
  - $!0 = 1$  and  $!x = 0, \forall x \neq 0$
  - Useful in C, no booleans
  - Some languages name this one differently

# Operations (on Integers)

- Left shift:  $x \ll y$  - move bits to the left
  - Effectively multiply by powers of 2
- Right shift:  $x \gg y$  - move bits to the right
  - Effectively divide by powers of 2
  - Signed (extend sign bit) vs unsigned (extend 0)

## Left Bit-shift Example

01011010 << 2

# Right Bit-shift Example

01011010 >> 3

# Bit-shift

Computing bit-shift effectively multiplies/divides by powers of 2

Consider decimal:

$$2130 \ll_{10} 2 = 213000 = 2130 \times 100$$

$$2130 \gg_{10} 1 = 213 = 2130 / 10$$

# Right Bit-shift Example 2

11001010 >> 1

# Right Bit-shift Example 2

For **signed** integers, extend the sign bit (1)

- Keeps negative value (if applicable)
- Approximates divide by powers of 2

11001010 >> 1

## Bit fiddling example