Bitwise Operations, Floating Point Numbers

CS 2130: Computer Systems and Organization 1

September 5, 2025

Announcements

- Quiz 1 opens this afternoon, due Sunday night
- Homework 1 due September 15

Two's Complement

Left Bit-shift Example

```
01011010 << 2
```

Right Bit-shift Example

```
01011010 >> 3
```

Bit-shift

Computing bit-shift effectively multiplies/divides by powers of 2

Consider decimal:

$$2130 \ll_{10} 2 = 213000 = 2130 \times 100$$

$$2130 \gg_{10} 1 = 213 = 2130 \ / \ 10$$

Right Bit-shift Example 2

`11001010 >> 1`

Right Bit-shift Example 2

For **signed** integers, extend the sign bit (1)

- Keeps negative value (if applicable)
- Approximates divide by powers of 2

```
11001010 >> 1
```

Bit fiddling example

What about other kinds of numbers?

Non-Integer Numbers

Floating point numbers

- Decimal: 3.14159

Non-Integer Numbers

Floating point numbers

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- Binary: 11.10110

Non-Integer Numbers

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- With integers, the point is always fixed after all digits
- With floating point numbers, the point can move!

Non-Integer Numbers

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Challenge! only 2 symbols in binary

Scientific Notation

Convert the following decimal to scientific notation:

$$2130$$

Scientific Notation

Convert the following binary to scientific notation:

$$101101$$

Something to Notice

An interesting phenomenon:

- Decimal: first digit can be any number *except* 0

$$2.13 \times 10^3$$

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$$1.01101 \times 2^5$$

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- Binary: first digit can be any number *except* 0 Wait!

$$1.01101 \times 2^5$$

  - First digit can only be 1

Floating Point in Binary

We must store 3 components

- **sign** (1-bit): 1 if negative, 0 if positive
- **fraction** or **mantissa**: (?-bits): bits after binary point
- **exponent** (?-bits): how far to move binary point

We do not need to store the value before the binary point. Why?

Floating Point in Binary

How do we store them?

- Originally many different systems
- IEEE standardized system (IEEE 754 and IEEE 854)
- Agreed-upon order, format, and number of bits for each

$$1.01101 \times 2^5$$

Example

A rough example in Decimal:

$$6.42 \times 10^3$$

Exponent

How do we store the exponent?

- Exponents *can* be negative

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

- Need positive and negative ints (but no minus sign)

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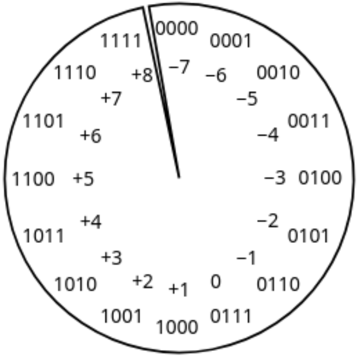
- Biased integers
  - Make comparison operations run more smoothly
  - Hardware more efficient to build
  - Other valid reasons

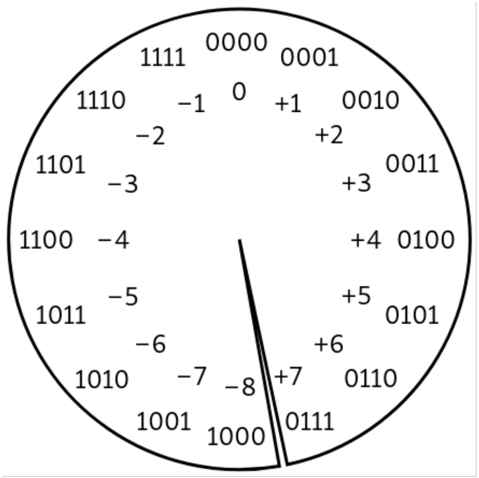
Biased Integers

Similar to Two's Complement, but add **bias**

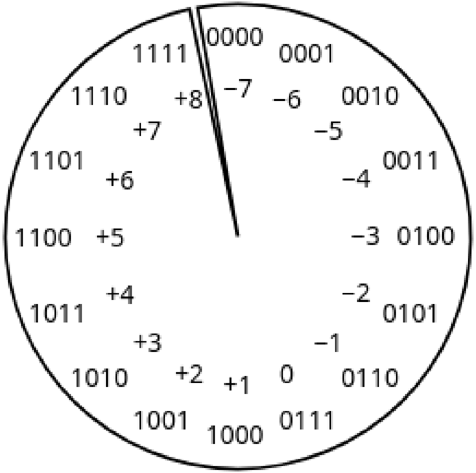
- **Two's Complement**: Define 0 as 00...0
- **Biased**: Define 0 as 0111...1
- Biased wraps from 000...0 to 111...1



Biased Integers



Two's Complement



Biased

Biased Integers Example

Calculate value of biased integers (4-bit example)

0010

Biased Integers

Floating Point Example

$$101.011_2$$

Floating Point Example

$$101.011_2$$

Floating Point Example

What does the following encode?

1	001110	1010101

Floating Point Example

What does the following encode?

1	001110	1010101

What about 0?

Floating Point Numbers

Four cases:

- **Normalized**: What we have seen today

$$s\ eeee\ ffff = \pm 1.ffff \times 2^{eeee-\text{bias}}$$

- **Denormalized**: Exponent bits all 0

$$s\ eeee\ ffff = \pm 0.ffff \times 2^{1-\text{bias}}$$

- **Infinity**: Exponent bits all 1, fraction bits all 0 (i.e., $\pm\infty$)
- **Not a Number (NaN)**: Exponent bits all 1, fraction bits not all 0