NP-Completeness

CS 4102: Algorithms

Spring 2022

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Today’s Keywords

- Reductions
- P vs NP
- NP Hard, NP Completeness
- k-Independent Set
- k-Vertex Cover
- 3SAT
- k-Clique

Why Study NP-Completeness

- All semester, we've studied ***finding algorithms*** to solve problems using various tools.
- Sometimes we instead need to prove that a problem is ***extremely hard***, so as not to waste time on it!
  - NP-Complete Problems are hard
  - Let's go over a few of them quickly
  - Let's show how to prove a new problem is NP-Complete

Some Preliminaries

Before we go further on this topic….

- This is a complex (and interesting!) topic in CS theory
- In our few lectures, we may approach things from a simpler viewpoint than you'd get in a CS theory course

- The math and theory related to NP-complete problems starts with ***decision problems***
  - What's that? Let's use vertex cover as an example
  - What's described next applies to any optimization problems we've seen

Forms of the Vertex Cover Problem

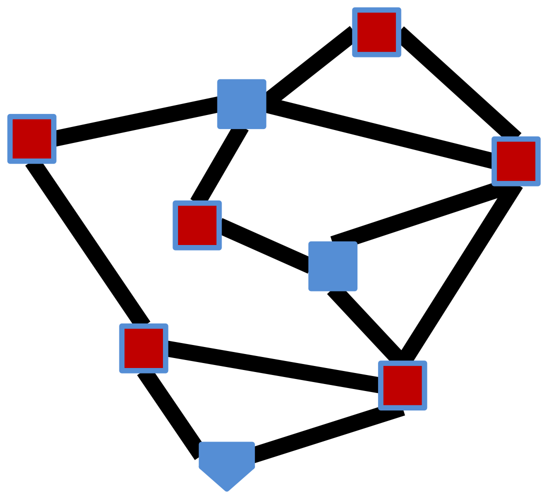
Vertex Cover: $C \subseteq V$ is a vertex cover if every edge in E has one of its endpoints in C

- **Minimum Vertex Cover Problem:** Given a graph $G = (V, E)$ find the minimum vertex cover C
  - Result is C, a set of vertices
- **k Vertex Cover Problem:** Given a graph $G = (V, E)$ and an integer k, **determine if there is a vertex cover C of size k**
  - Result is True or False
  - This is the ***decision problem form*** of Vertex Cover

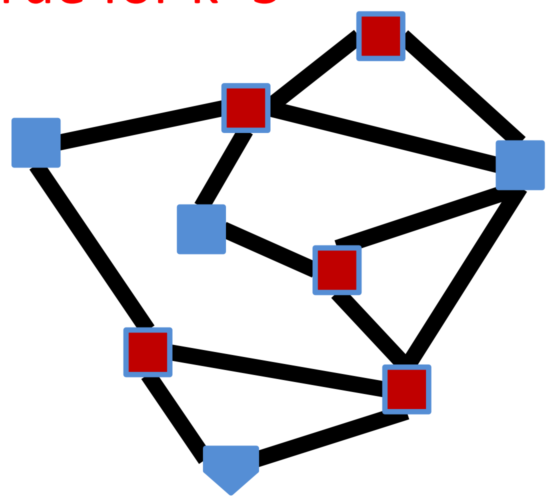
k Vertex Cover

- k **Vertex Cover Problem:** Given a graph $G = (V, E)$ and an integer k, **determine if there is a vertex cover C of size k**

True for k=6



True for k=5



Is 5 the smallest?

True for k=4?

Problem Types

- **Decision Problems:**
  - Is there a solution?
    - Result is True/False
  - E.g. Is there a vertex cover of size k?
- **Optimal Value Problems:**
  - E.g. What's the min *k* for *k*-vertex cover decision problem?
- **Search Problems:**
  - Find a solution
    - Result more complex than T/F or a k
  - E.g. Find a vertex cover of size k
- **Verification Problems:**
  - Given a potential solution for an input, is that input valid?
    - Result is True/False
    - For *decision problem*, check solution to its *search problem*
  - E.g. Is **set of vertices** a vertex cover of size k**?**

If we can solve this…

…Then we can solve this…

…and also this

Looking ahead:
We'll use this to define a problem classes P and NP

Looking ahead:
We'll use this to define a problem class called NP

Using a k-VertexCover decider to build a searcher

Note this is a reduction!
kVC-search $\le_p$ kVC-decider

- Set $i = k - 1$
- Remove nodes (and incident edges) one at a time
- Check if there is a vertex cover of size i (i.e. use the "decider")
  - If so, then that removed node was part of the k vertex cover, set $i = i - 1$
  - Else, it wasn't

Did I need this node to cover its edges to have a vertex cover of size k?

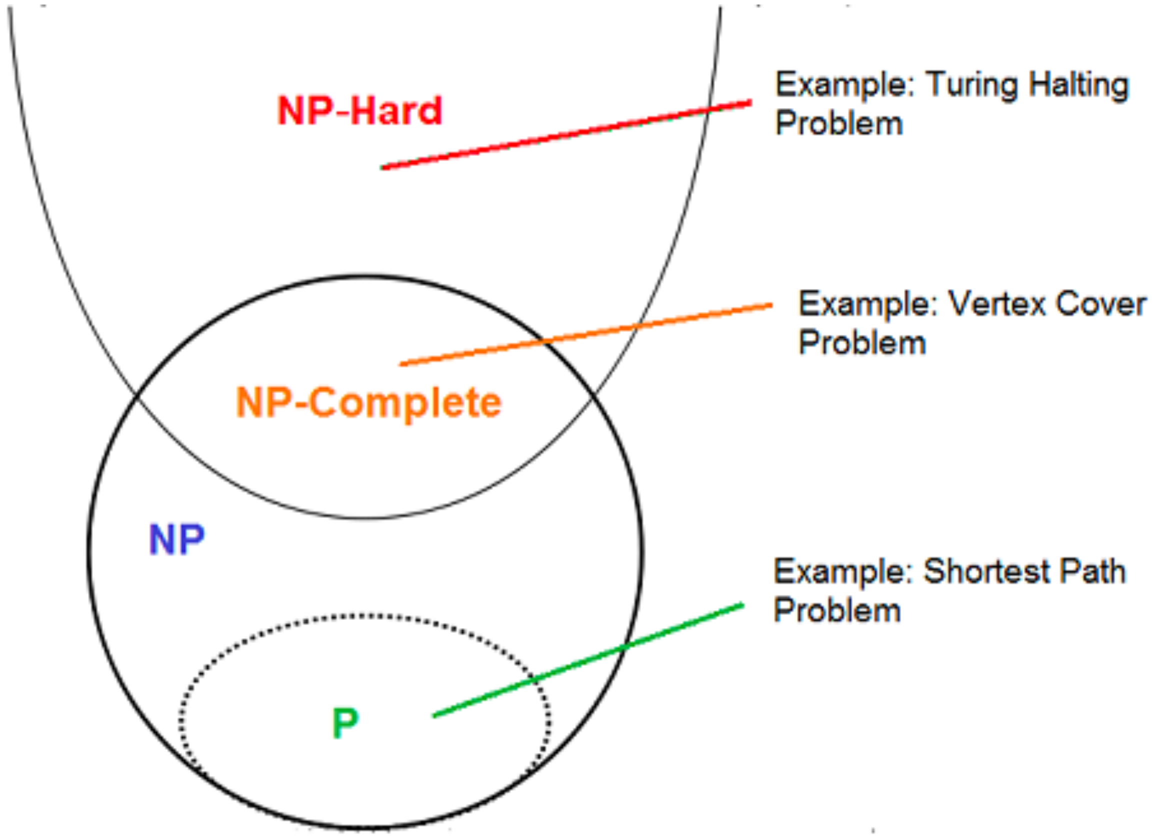
Quick Background!

- ***P***: Set of problems solved in polynomial time (e.g., sorting a list)

- ***NP***: Set of problems that can be:
  1) Solved in non-deterministic polynomial time
  2) A solution verified in polynomial time

- ***NP-Hard***: Set of problems that are as hard as (or harder) than the hardest problems in NP

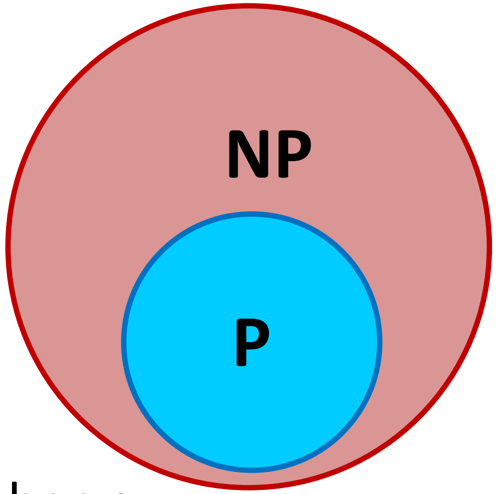
- ***NP-Complete***: Set of problems that are both NP and NP-Hard (i.e., the equally hardest problems in NP)



This diagram assumes that P != NP

Classes of Problems: P vs NP

- P
  - Deterministic Polynomial Time
  - P is the set of problems solvable in polynomial time
    - $O(n^c)$ for some number c
- NP
  - Non-Deterministic Polynomial Time
  - NP is the set of problems ***verifiable*** in polynomial time
    - Verify a proposed solution (not find one) in $O(n^c)$ for some number c
    - For decision problems, really verifying using some information we call a *certificate*
- Open Problem: Does P=NP?
  - Certainly $P \subseteq NP$



k-Independent Set is NP

To show: Given a potential solution S, can we **verify** it in $O(n^c)$? $[n = V + E]$

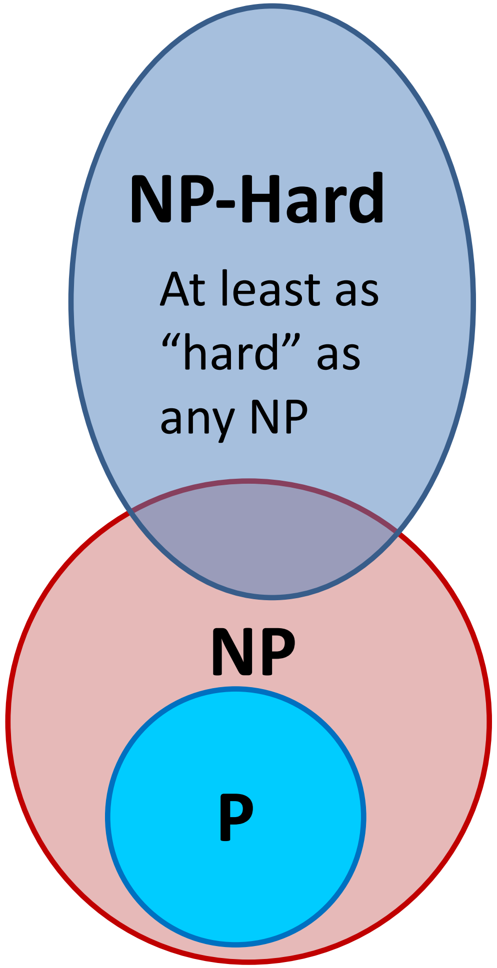
How can we verify it?

1. Check that S is of size k? Takes $O(V)$
2. Check that S is an independent set? Takes $O(V^2)$

Therefore, *k-IndSet* $\subseteq \boldsymbol{NP}$

NP-Hard

- How can we try to figure out if P=NP?
- Identify problems at least as "hard" as any NP
  - If any of these "hard" problems can be solved in polynomial time, then all NP problems can be solved in polynomial time.
- Definition: NP-Hard:
  - B is NP-Hard if $\forall A \in NP, A \leq_p B$
  - $A \leq_p B$ means A reduces to B in polynomial time
  - Remember: $A \leq_p B$ implies A is not harder than B



Polynomial Reduction & Relative Hardness

$\boxed{A \leq_p B}$

A problem we <u>don't</u> know how to solve

A

Then A could be solved in polynomial time

Solution for $\boldsymbol{A}$

R_A

Polynomial Reduction

Map Instances of problem $\boldsymbol{A}$ to Instances of $\boldsymbol{B}$

Map Solutions of problem $\boldsymbol{B}$ to Solutions of $\boldsymbol{A}$

A problem we <u>do</u> know how to solve

B

Using any Algorithm for $\boldsymbol{B}$

If B could be solved in polynomial time

Solution for $\boldsymbol{B}$

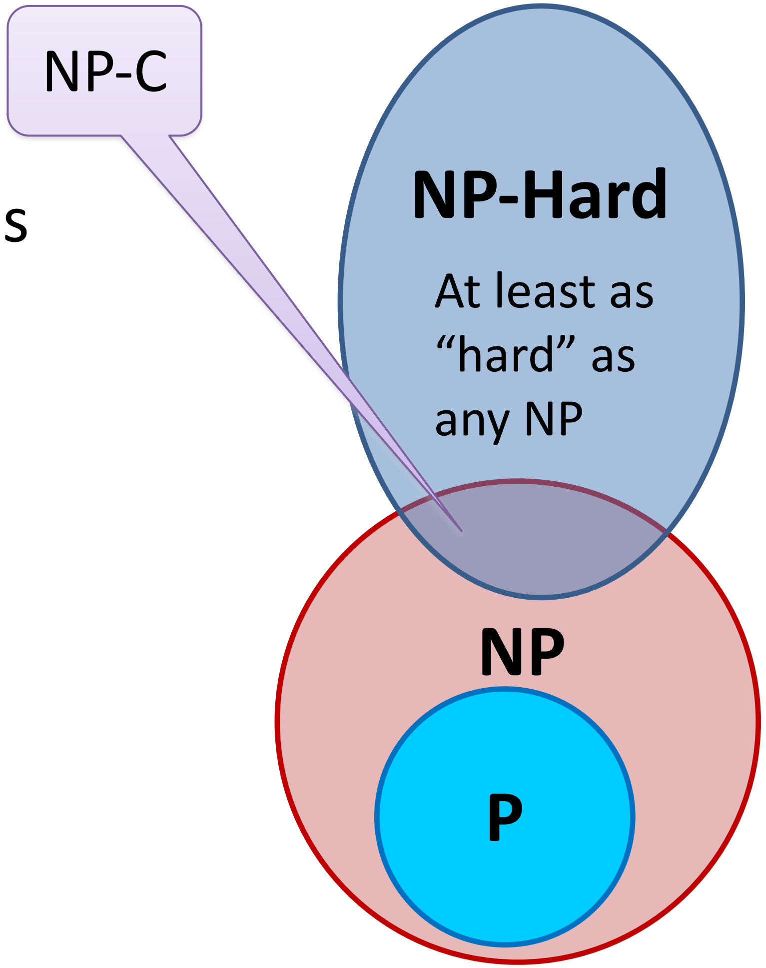
R_B

NP-Complete

NP-Complete = NP ∩ NP-Hard

- The "hardest" of all the problems in NP
- An NP-C problem is polynomial iff all NP problems are polynomial. I.e. P=NP
- If P=NP, then all NP-C problems are polynomial
- "Together they stand, together they fall"

- **How to show a problem C is NP-Complete?**
  - Show C belongs to NP
    - Show we can verify a solution in polynomial time
  - Show C is NP-Hard
    - $\forall A \in NP, A \leq_p C$ (That sounds really hard to do!)
    - Or, show a reduction from another NP-Hard problem. (Why? Details next.)



NP-Completeness

- So…a problem is NP-Complete if you can do the following:

- 1) Show how to verify it in polynomial time
  - Given a solution to the problem, verify it is correct
  - That algorithm's runtime needs to be a polynomial (usually easy)

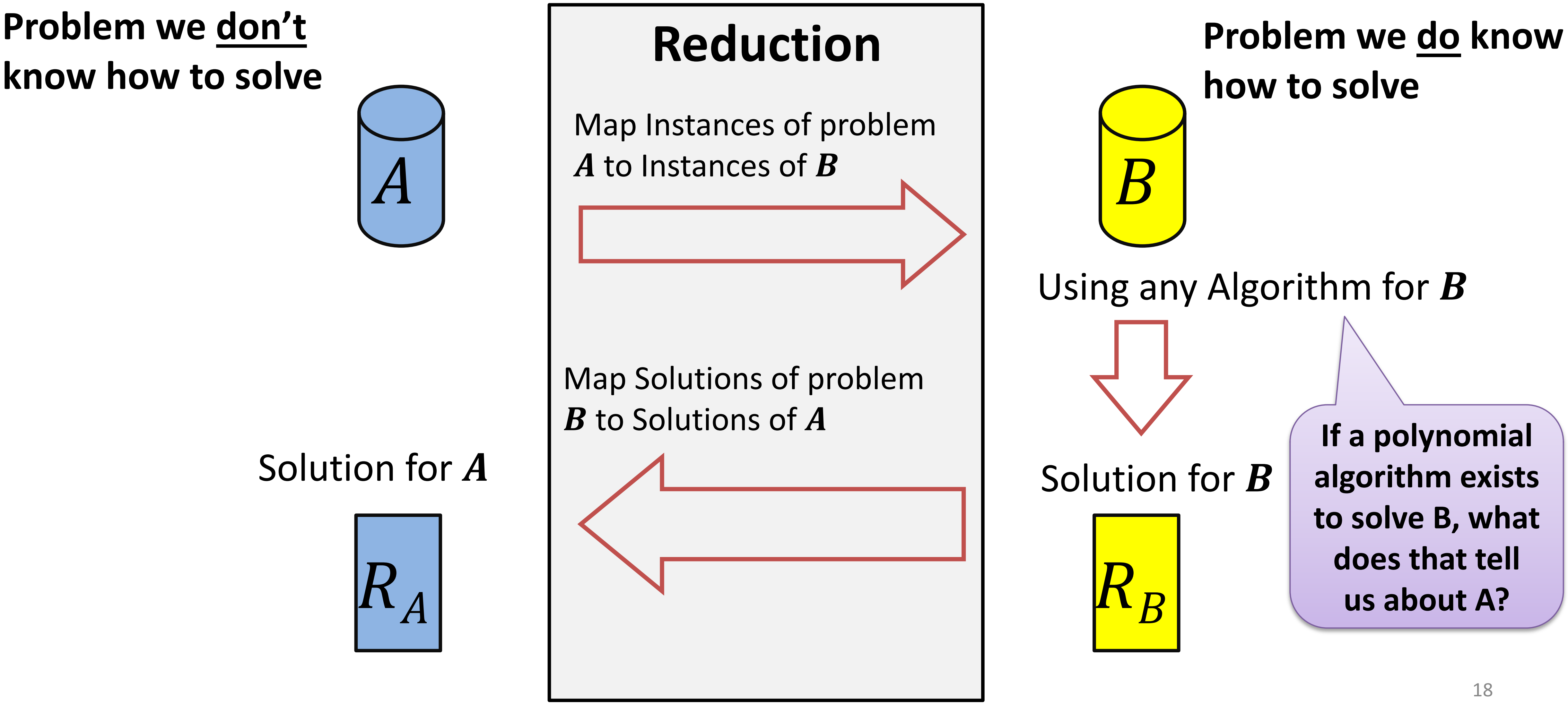
- 2) Show the problem is NP-Hard (as hard or harder than a known NP-Hard Problem)
  - Take a currently known NP-Hard problem (let's call it A)
  - Show that $A \leq_p X$ (where X is your problem)
  - Why? If A is NP-Hard, then: $\quad$ *any NP problem* $\leq_p A$
  - Transitivity: $\quad$ *any NP problem* $\leq_p A \leq_p X$
  - So X satisfies definition of NP-Hard

“Consequences” of NP-Completeness

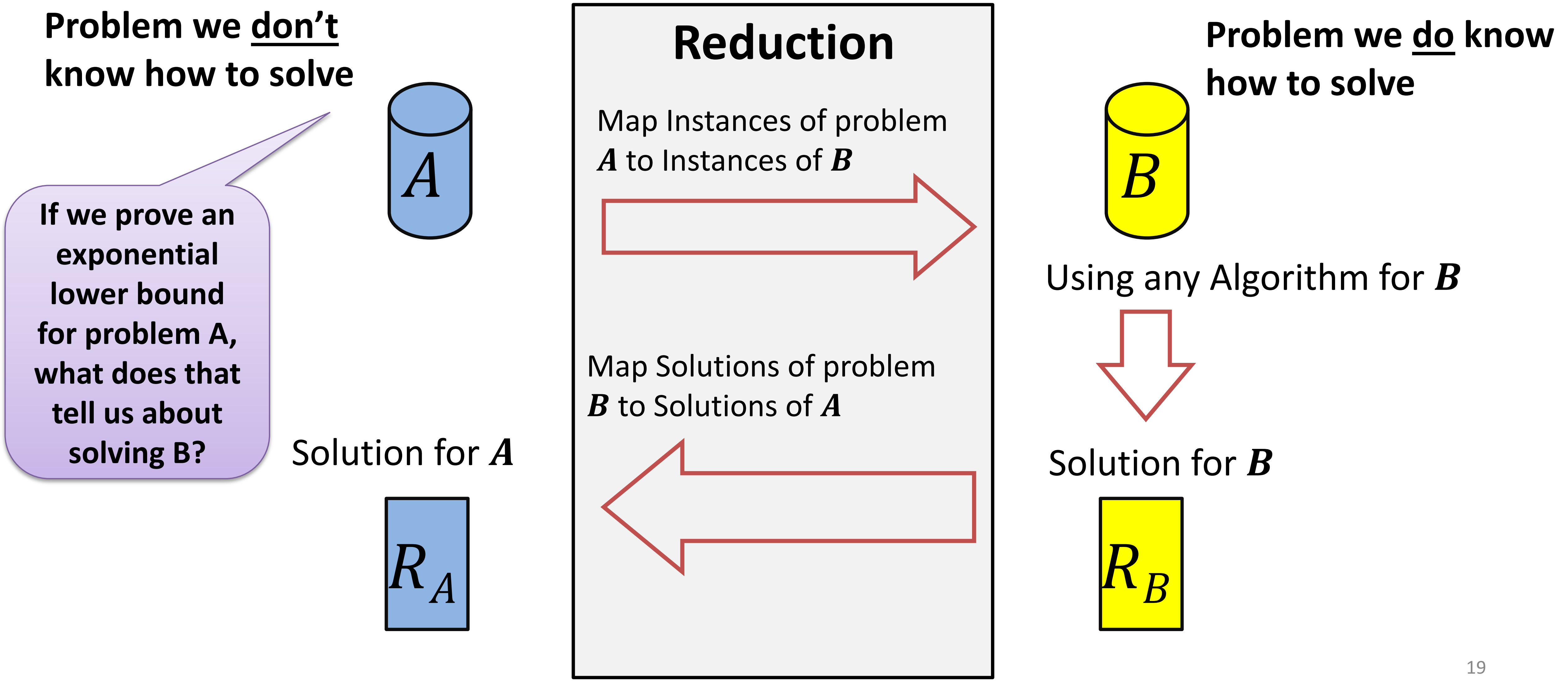
- NP-Complete is the set of “hardest” problems in NP, with these important properties:
  - If any *one* NP-Complete problem can be solved in polynomial time…
  - …then *every* NP-Complete problem can be solved in polynomial time…
  - …and in fact *every* problem in **NP** can be solved in polynomial time (which would show **P = NP**)
  - Or, prove an exponential lower-bound for *any single* **NP-hard** problem, then *every* **NP-hard** problem (including **NP-C**) is exponential

Therefore: solve (say) traveling salesperson problem in O(n^{100}) time, you've proved that **P = NP**. Retire rich & famous!

$A \le_p B$ and B in P



$A \leq_p B$ and we prove A not in P



Summary of Where We Are

- Focusing on “hard” problems, those that seem to be exponential
- Reductions used to show “hardness” relationships between problems
- Starting to define “classes” of problems based on complexity issues
  - P are problems that can be solved in polynomial time
  - NP are problems where a solution can be verified in polynomial time
  - NP-hard are problems that are at least as hard as anything in NP
  - NP-complete are NP-hard problems that “stand or fall together”

Review: P And NP Summary

- **P** = set of problems that can be solved in polynomial time
- **NP** = set of problems for which a solution can be verified in polynomial time
  - Note: this is a more “informal” definition, but it’s fine for CS4102
  - See later slide on ”certificates” for more info.
- **P** $\subseteq$ **NP**
- Open question: Does **P = NP**?

More Reminders and Some Consequences

- **Definition of NP-Hard and NP-Complete:**
  - If all problems A $\in$ **NP** are reducible to B , then B is *NP-Hard*
  - We say B is *NP-Complete* if:
    - B is NP-Hard
    - and B $\in$ **NP**
- Any NP-C must reduce to any other NP-C. Can you see why?
- If B $\leq_p$ C and B is NP-Complete, C is also NP-Complete
  - Don't see why? We'll show details in two more slides
  - As long as C $\in$ **NP**. Otherwise can only say C $\in$ **NP-hard.**

But You Need One NP-Hard First…

- If you have one NP-Hard problem, you can use the technique just described to prove other problems are NP-Hard and NP-c
  - We need an NP-C problem to start this off
- The definition of NP-Hard was created to prove a point
  - There *might be* problems that are at least as hard as "anything" (i.e. all NP problems)
- Are there really NP-complete problems?
- **Cook-Levin Theorem: The satisfiability problem (SAT) is NP-Complete.**
    - Stephen Cook proved this "directly", from first principles, in 1971
    - Proven independently by Leonid Levin (USSR)
    - Showed that any problem that meets the definition of NP can be transformed in polynomial time to a CNF formula.
    - Proof outside the scope of this course (lucky you)

More About The SAT Problem

- The first problem to be proved NP-Complete was *satisfiability* (SAT):
  - Given a Boolean expression on *n* variables, can we assign values such that the expression is TRUE?
  - Ex: $((x_1 \rightarrow x_2) \vee \neg((\neg x_1 \leftrightarrow x_3) \vee x_4)) \wedge \neg x_2$
- You might imagine that lots of decision problems could be expressed as a complex logical expression
  - And Cook and Levin proved you were right!
  - Proved the general result that any NP problem can be expressed this way

Conjunctive Normal Form (CNF)

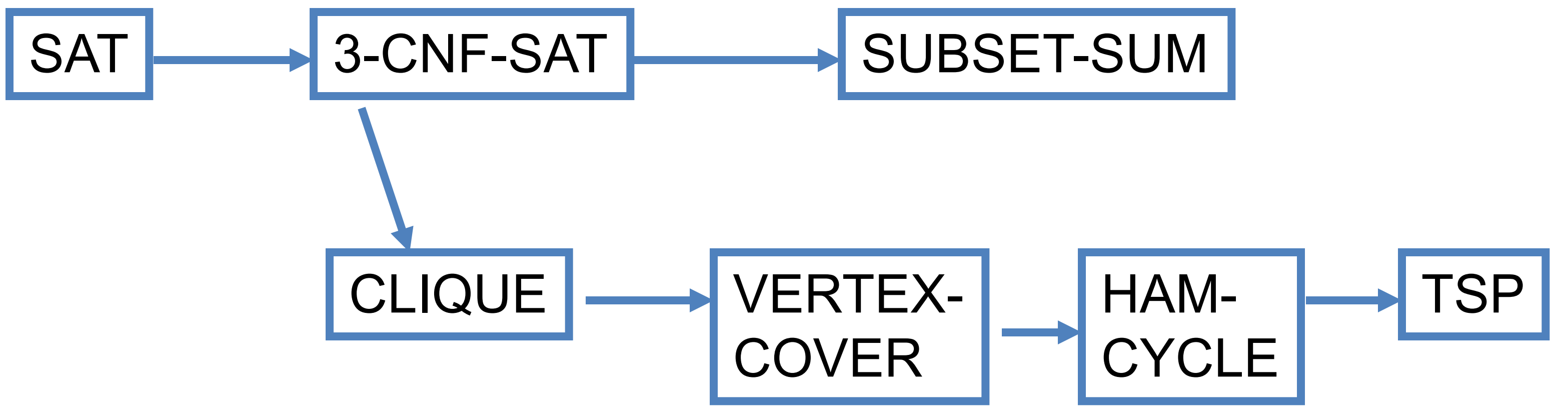
- Even if the form of the Boolean expression is simplified, the problem may be NP-Complete
  - *Literal*: an occurrence of a Boolean or its negation
  - A Boolean formula is in *conjunctive normal form*, or *CNF*, if it is an AND of clauses, each of which is an OR of literals
    - Ex: $(x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_3 \vee x_4) \wedge (\neg x_5)$
  - *3-CNF*: each clause has exactly 3 distinct literals
    - Ex: $(x_1 \vee \neg x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_3 \vee x_4) \wedge (\neg x_5 \vee x_3 \vee x_4)$
    - Notice: true if at least one literal in each clause is true
  - Note: Arbitrary SAT expressions can be translated into CNF forms by introducing intermediate variables etc.

The 3-CNF Problem

- Satisfiability of Boolean formulas in 3-CNF form (the *3-CNF Problem*) is NP-Complete
  - Intuitively it's not hard to imagine that SAT $\leq_p$ 3-CNF
  - Proof: Also done by Cook ("part 2" of Cook's theorem)
- The reason we care about the 3-CNF problem is that it is relatively easy to reduce to others
  - Having a proof that 3-CNF is NP-Complete lets us use it to prove many seemingly unrelated problems are NP-Complete

Joining the Club

- Given one NP-c problem, others can join the club
  - Prove that SAT reduces to another problem, and so on…



  - Membership in NP-c grows…
  - Classic textbook: Garey, M. and D. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*, 1979.

Reductions to Prove NP-C

- Next:
  - A tour of how to prove some problems are NP-C
  - 3-SAT is a good starting point!

  - k-Clique
  - k-Independent Set
  - k-Vertex Cover

Reminder about 3-SAT

- Shown to be NP-hard by Cook
- Given a 3-CNF formula (logical AND of clauses, each an OR of 3 variables), is there an assignment of true/false to each variable to make the formula true (i.e., <u>satisfy</u> the formula)?

$$\underbrace{(x \vee y \vee z)}_{\text{Clause}} \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z}) \wedge (z \vee \bar{x} \vee u) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

Variables

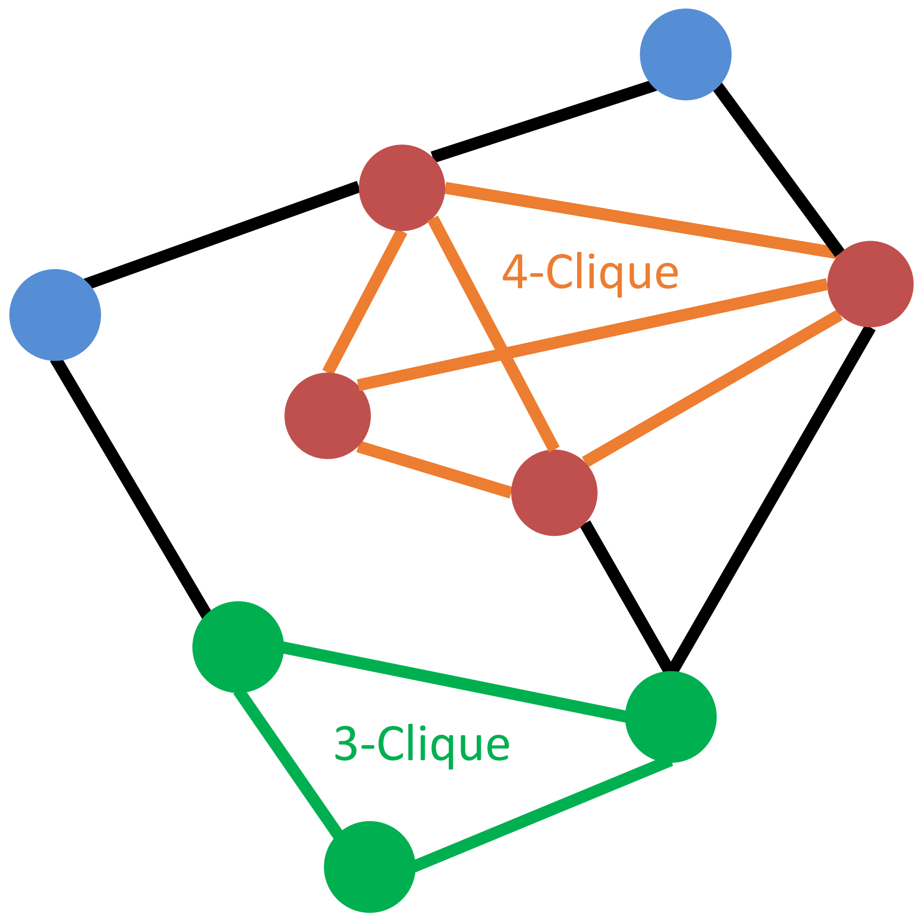
$x = \text{true}$
$y = \text{false}$
$z = \text{false}$
$u = \text{true}$

- Next example: *k*-Clique

- Let’s show that k-Clique is NP-Complete!

k-Clique Problem

- **Clique:** A complete subgraph
- **k-Clique problem:** given a graph G and a number k, is there a clique of size k?

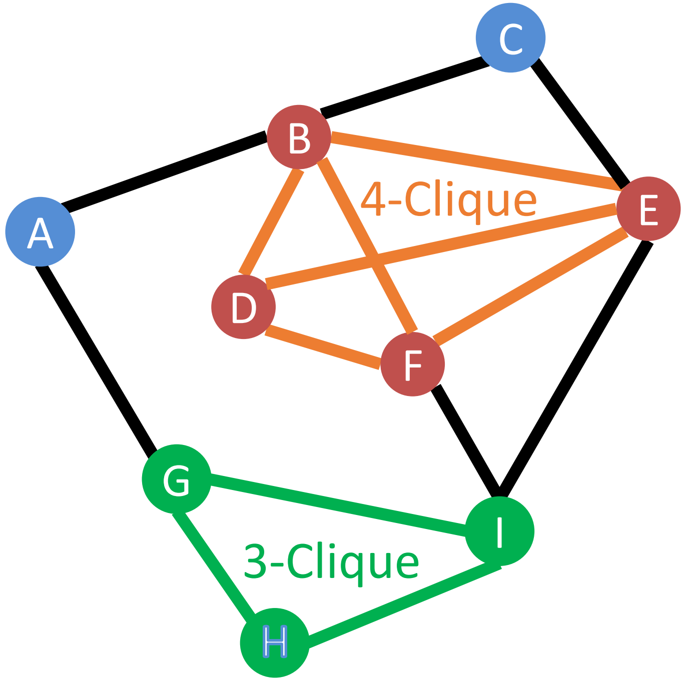


k-Clique is NP-Complete

1. Show that it belongs to NP
   – Give a polynomial time verifier
2. Show it is NP-Hard
   – Give a reduction from a known NP-Hard problem
   – We will show 3-SAT $\leq_p$ k-clique

k-Clique is in NP

- **Show:** For any graph G:
  - There is a short certificate ("solution") that G has a k-clique
  - The certificate can be checked efficiently (in polynomial time)



Graph G

Suppose $k = 4$

Certificate for G: $S = \{B, D, E, F\}$
(nodes in the k-clique)

Checking the certificate:

- Check that $|S| = k$ $\quad O(k) = O(|V|)$
- Check that every pair of nodes in S share an edge $\quad O(k^2) = O(|V|^2)$

Total time: $O(|V|^2) = \text{poly}(|V| + |E|)$

k-Clique is NP-Complete

1. Show that it belongs to NP ✅
   – Give a polynomial time verifier
2. Show it is NP-Hard
   – Give a reduction from a known NP-Hard problem
   – We will show 3-SAT $\leq_p$ k-clique

3-SAT $\le_p$ $\boldsymbol{k}$-Clique

3-SAT

$(x \lor y \lor z) \land (x \lor \bar{y} \lor y) \land (u \lor y \lor \bar{z}) \land$
$(z \lor \bar{x} \lor u) \land (\bar{x} \lor \bar{y} \lor \bar{z})$

$x = \text{true}$
$y = \text{false}$
$z = \text{false}$
$u = \text{true}$

Map instances of problem $\boldsymbol{A}$ to instances of $\boldsymbol{B}$

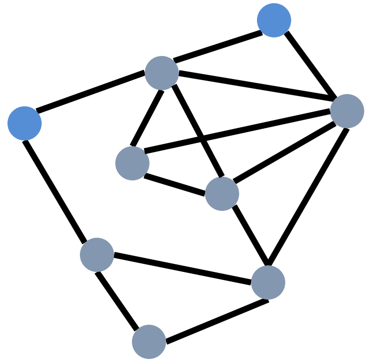
polynomial time

Map solutions of problem $\boldsymbol{B}$ to solutions of $\boldsymbol{A}$

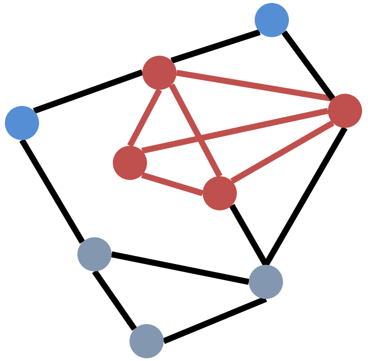
polynomial time

polynomial-time reduction

k-clique



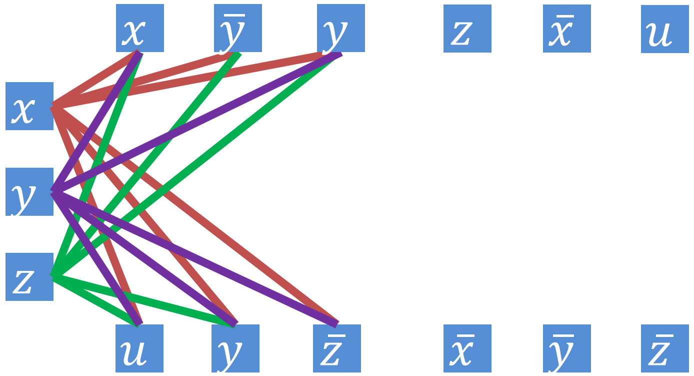
value k



3-SAT $\leq_p$ $\boldsymbol{k}$-Clique

$$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z}) \wedge (z \vee \bar{x} \vee u) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

$k = 5$



(also do this for the other clauses, omitted due to clutter)

For each clause, introduce a node for each of its three variables

Add an edge from each node to all non-contradictory nodes in the other clauses (i.e., to all nodes that is not the negation of its own variable)

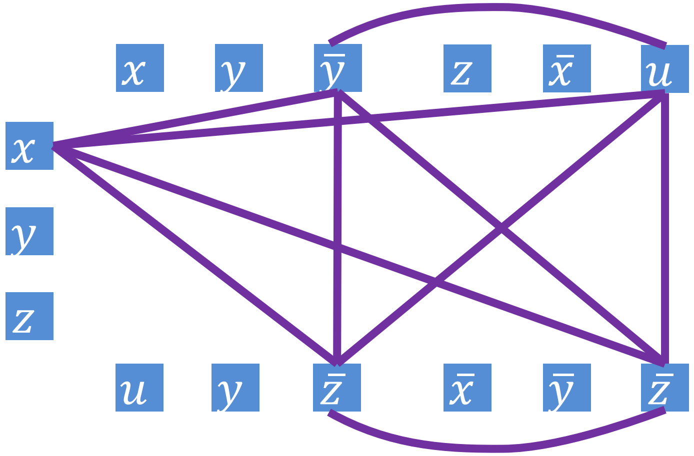
Let k = number of clauses

Claim. There is a k-clique in this graph if and only if there is a satisfying assignment

3-SAT $\leq_p$ $\boldsymbol{k}$-Clique

$$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z}) \wedge (z \vee \bar{x} \vee u) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

$k = 5$



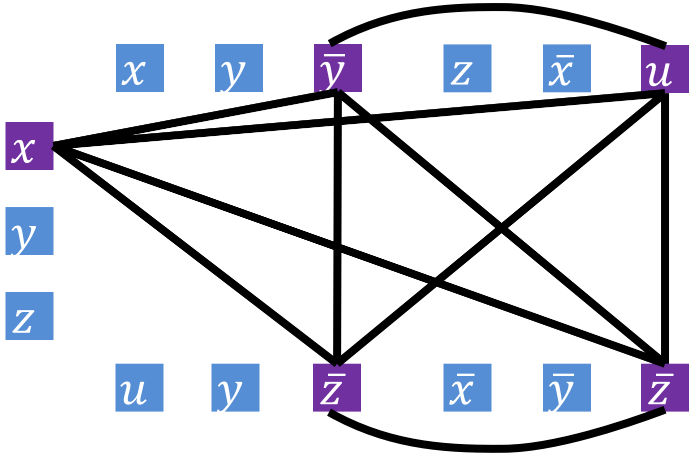
Suppose there is a k-clique in this graph

- There are no edges between nodes for variables in the same clause, so k-clique must contain one node from each clause
- Nodes in clique cannot contain variable and its negation
- Nodes in clique must then correspond to a satisfying assignment

3-SAT $\leq_p$ $\boldsymbol{k}$-Clique

$$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z}) \wedge (z \vee \bar{x} \vee u) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

$k = 5$



x = true
y = false
z = false
u = true

Suppose there is a satisfying assignment to the formula

- For each clause, choose one node whose value is true
- There are k clauses, so this yields a collection of k nodes
- Since the assignment is consistent, there is an edge between every pair of nodes, so this constitutes a k-clique

3-SAT $\leq_p$ $\boldsymbol{k}$-Clique

3-SAT

$(x \lor y \lor z) \land (x \lor \bar{y} \lor y) \land (u \lor y \lor \bar{z}) \land$
$(z \lor \bar{x} \lor u) \land (\bar{x} \lor \bar{y} \lor \bar{z})$

$x = \text{true}$
$y = \text{false}$
$z = \text{false}$
$u = \text{true}$

Map instances of problem $\boldsymbol{A}$ to instances of $\boldsymbol{B}$

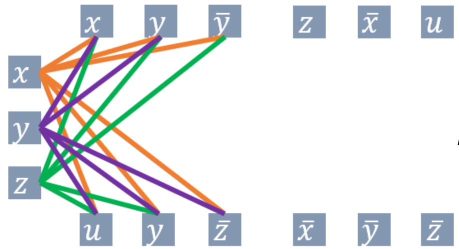
polynomial time

Map solutions of problem $\boldsymbol{B}$ to solutions of $\boldsymbol{A}$

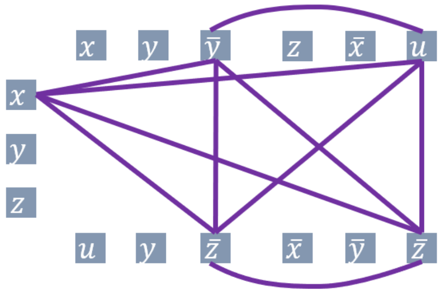
polynomial time

polynomial-time reduction

k-clique



$k = 5$



$k = 5$

k-Clique is NP-Complete

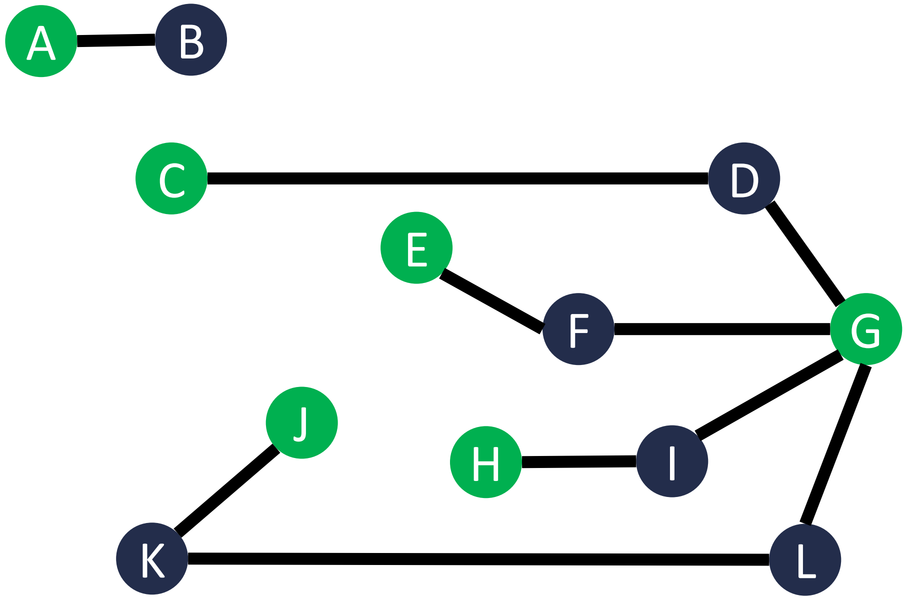
1. Show that it belongs to NP
   – Give a polynomial time verifier
2. Show it is NP-Hard
   – Give a reduction from a known NP-Hard problem
   – We will show 3-SAT $\leq_p$ k-clique

k-Independent Set is NP-Complete

1. Show that it belongs to NP
2. Show it is NP-Hard
   - Show 3-SAT $\leq_p$ k-Independent Set

k-Independent Set is in NP

- **Show:** For any graph G:
  - There is a short **certificate** ("solution" for search problem) that G has a k-independent set
  - The certificate can be checked efficiently (in polynomial time)



Graph G

Certificate for G: $S = \{A, C, E, G, H, J\}$
(nodes in the k-independent set)

Checking the certificate:

- Check that $|S| = k$ $\quad O(k) = O(|V|)$
- Check that every edge is incident on at most one node in S $\quad O(|V| + |E|)$

Total time: $O(|E| + |V|) = \text{poly}(|V| + |E|)$

k-Independent Set is NP-Complete

1. Show that it belongs to NP ✅
2. Show it is NP-Hard
   – Show 3-SAT $\leq_p$ k-Independent Set

3-SAT $\leq_p$ $\boldsymbol{k}$-Independent Set

3-SAT

$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z})$

$x = \text{true}$
$y = \text{false}$
$z = \text{false}$
$u = \text{true}$

Map instances of problem $\boldsymbol{A}$ to instances of $\boldsymbol{B}$

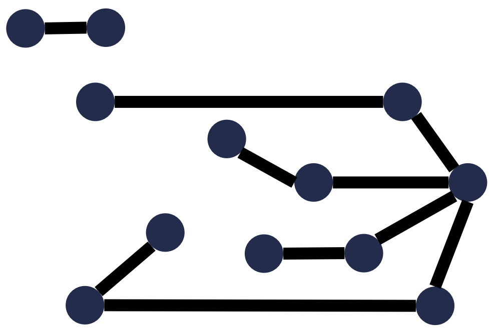
polynomial time

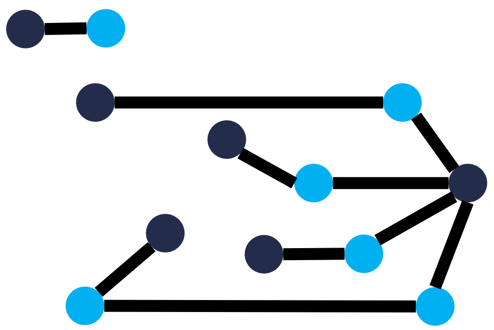
Map solutions of problem $\boldsymbol{B}$ to solutions of $\boldsymbol{A}$

polynomial time

polynomial-time reduction

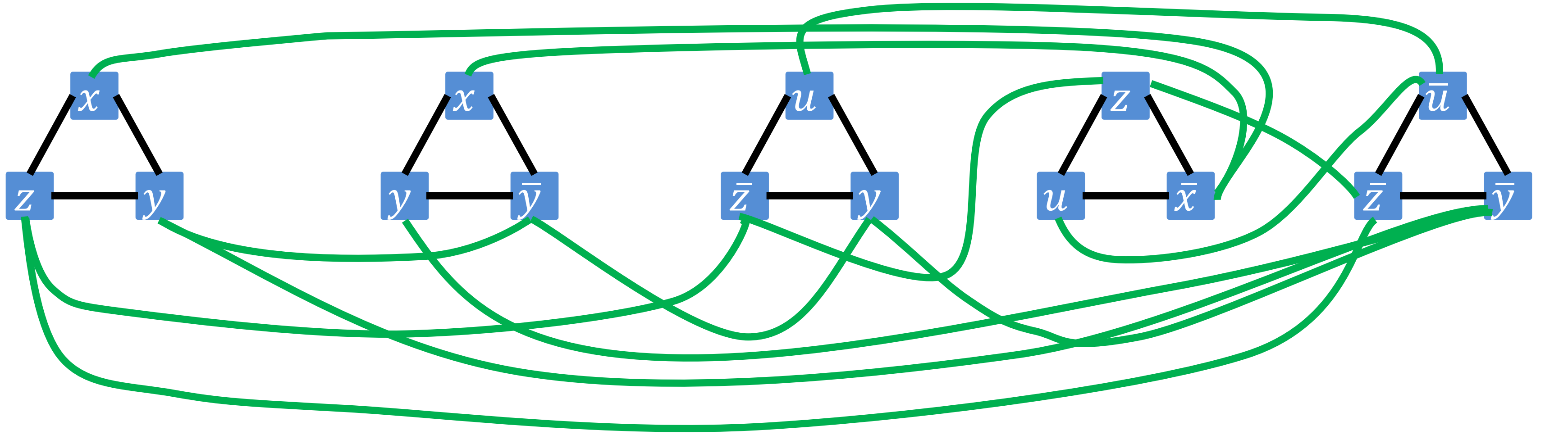
k-independent set





3-SAT $\leq_p$ $\boldsymbol{k}$-Independent Set

$$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z}) \wedge (z \vee \bar{x} \vee u) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$



For each clause, construct a <u>triangle graph</u> with its three variables as nodes

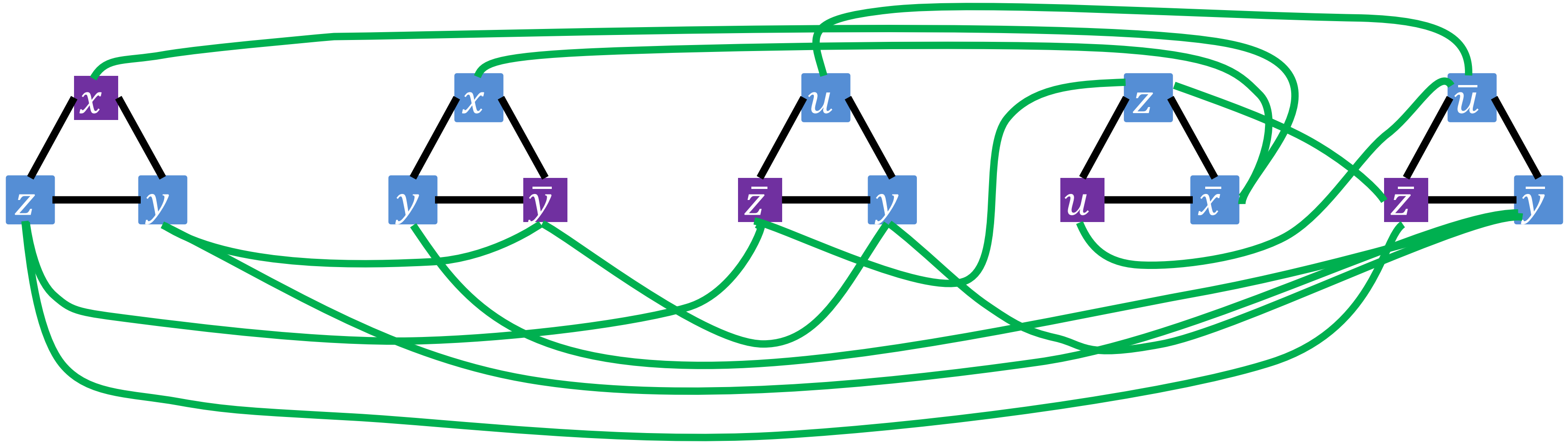
Add an edge between each node and its negation

Let k = number of clauses

Claim. There is a k-independent set in this graph if and only if there is a satisfying assignment

3-SAT $\leq_p$ $\boldsymbol{k}$-Independent Set

$$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z}) \wedge (z \vee \bar{x} \vee u) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$



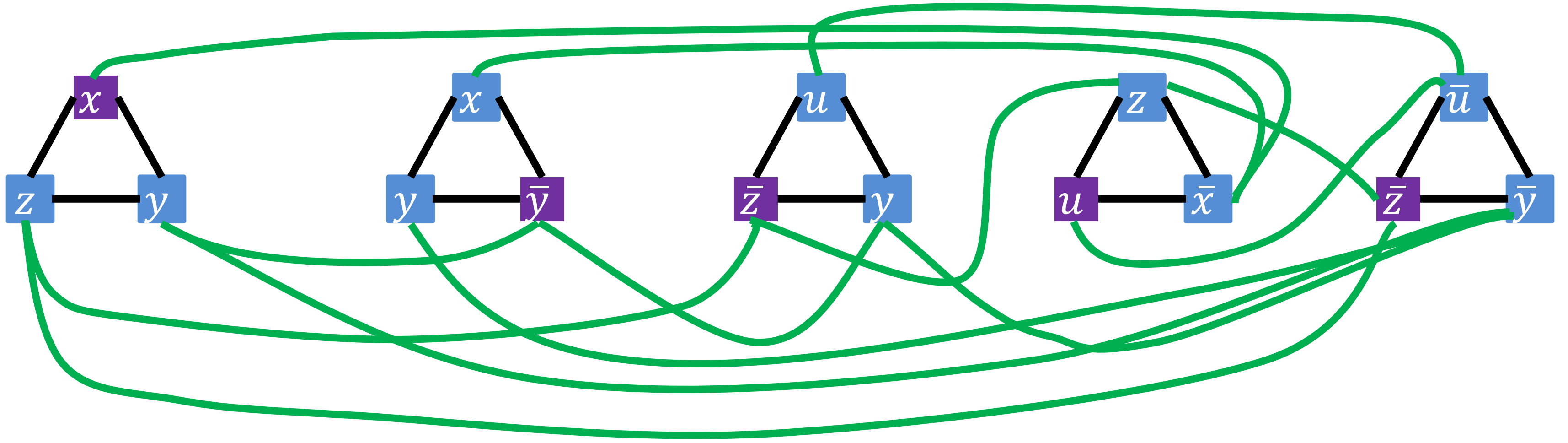
x = true
y = false
z = false
u = true

Suppose there is a k-independent set S in this graph G

- By construction of G, at most one node from each triangle is in S
- Since $|S| = k$ and there are k triangles, each triangle contributes one node
- If a variable x is selected in one triangle, then $\bar{x}$ is never selected in another triangle (since each variable is connected to its negation)
- There are no contradicting assignments, so can set variable chosen in each triangle to "true"; satisfying assignment by construction

3-SAT $\leq_p$ $\boldsymbol{k}$-Independent Set

$$(x \lor y \lor z) \land (x \lor \bar{y} \lor y) \land (u \lor y \lor \bar{z}) \land (z \lor \bar{x} \lor u) \land (\bar{x} \lor \bar{y} \lor \bar{z})$$



x = true
y = false
z = false
u = true

Suppose there is a satisfying assignment to the formula

- At least one variable in each clause must be true
- Add the node to that variable to the set S
- There are k clauses, so set S has exactly k nodes
- If we use x in any clause, we will never use $\bar{x}$, so there are no edges among the nodes in S

3-SAT $\leq_p$ $\boldsymbol{k}$-Independent Set

3-SAT

$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee y) \wedge (u \vee y \vee \bar{z})$

$x = \text{true}$
$y = \text{false}$
$z = \text{false}$
$u = \text{true}$

Map instances of problem $\boldsymbol{A}$ to instances of $\boldsymbol{B}$

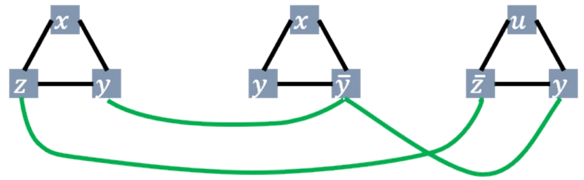
polynomial time

Map solutions of problem $\boldsymbol{B}$ to solutions of $\boldsymbol{A}$

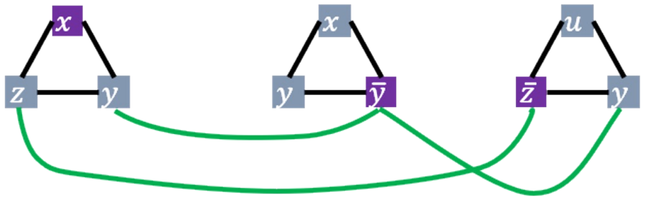
polynomial time

polynomial-time reduction

k-independent set



$k = 3$



$k = 3$

k-Independent Set is NP-Complete

1. Show that it belongs to NP ✅
2. Show it is NP-Hard ✅
   – Show 3-SAT $\leq_p$ k-independent set

- Next example: *k*-Vertex Cover

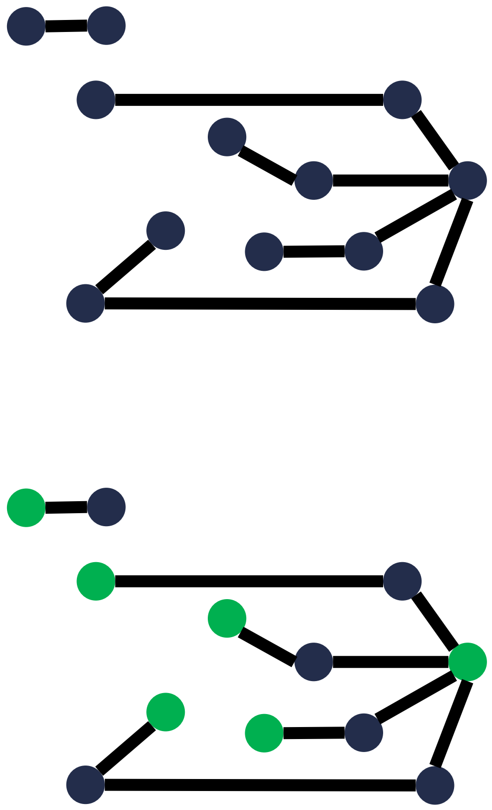
- Remember?
  - We did the following reduction in an earlier slide set!

    k-Independent Set $\leq_p$ *k*-Vertex Cover

  - We just showed *k*-Independent Set is NP-C
  - Therefore…. (you know, right?)

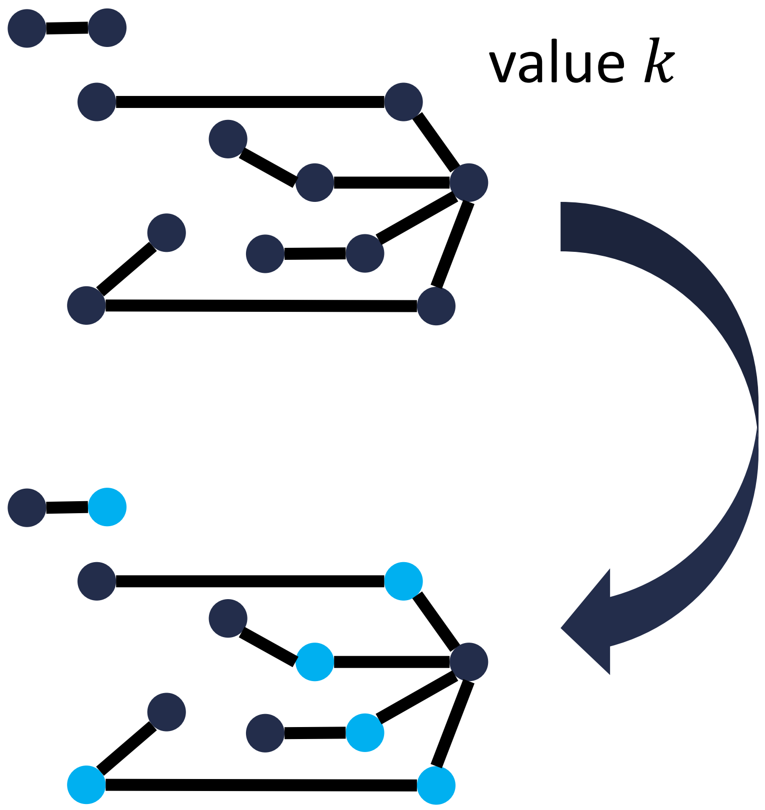
Max Independent Set $\leq_p$ k-Vertex Cover

k-independent set



k-vertex cover

value k



Map instances of problem $\boldsymbol{A}$ to instances of $\boldsymbol{B}$

$O(1)$ time

Map solutions of problem $\boldsymbol{B}$ to solutions of $\boldsymbol{A}$

$O(|V|)$ time

Reduction

k-Vertex Cover is NP-Complete

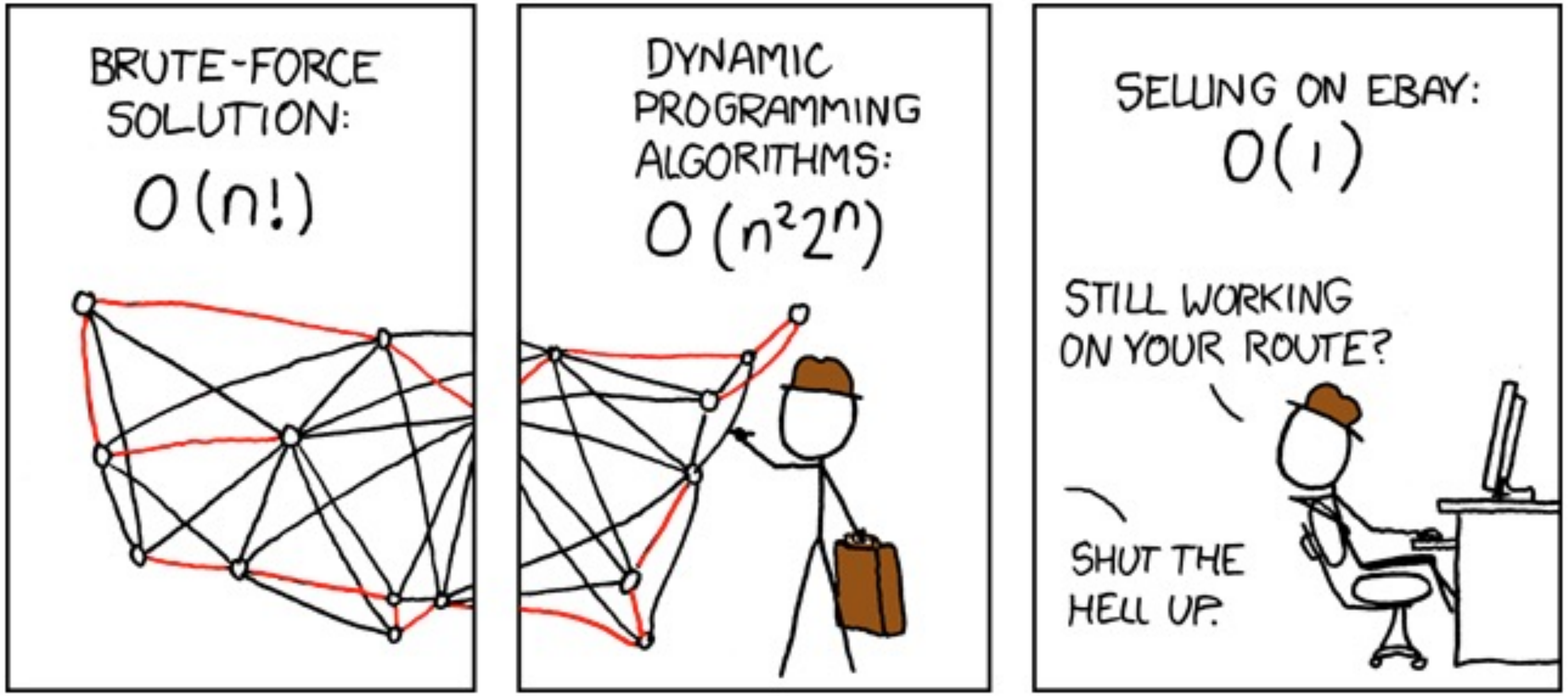
1. Show that it belongs to NP
   – Given a candidate cover, check that every edge is covered
2. Show it is NP-Hard
   – Show k-independent set $\leq_p$ k-vertex cover

Wrap Up and Reminders

Why Prove NP-Completeness?

- Though nobody has proven that **P ≠ NP**, if you prove a problem NP-Complete, most people accept that it is probably exponential
- Therefore it can be important for you to prove that a problem is NP-Complete
  - Don't need to try to come up perfect non-exponential algorithm
  - Can instead work on *approximation algorithms*

What's a poor salesperson to do?



http://xkcd.com/399/

Approximation Algorithms

- Look at first 3 pages of Ch. 35 of CLRS textbook
- Can we find an algorithm for problem A $\in$ **NP-C** that:
  - Runs in polynomial time
  - Gets "near optimal" results
- Prove some bound on the algorithm's correctness in terms of the true optimal result
  - No worse that (some factor) of optimal
  - "It's not always right (best), but it's guaranteed to be this close."

General Comments

- At least 3000 problems have been shown to be NP-Complete
  - That number is from a non-recent report, so we might say that counts is a weak lower-bound on the true number found
  - https://en.wikipedia.org/wiki/List_of_NP-complete_problems including some popular games
- Some reductions are profound, some are comparatively easy, many are easy once the key insight is given

Other NP-Complete Problems

- *Hamilton Path/Cycle, Traveling Salesperson*
- *Subset-sum*: Given a set of integers, does there exist a subset that adds up to some target *T* ?
- *0-1 knapsack*: when weights not just integers
- *Graph coloring*: can a given graph be colored with *k* colors such that no adjacent vertices are the same color?
- Etc…

Review (Again)

- A problem B is *NP-complete*
  - if it is in NP **and** it is NP-hard.
- A problem B is *NP-hard*
  - if *every* problem in NP is reducible to **B**.
- A problem A is *reducible* to a problem B if
  - there exists a polynomial reduction function T such that
    - For every string x,
    - if x is a yes input for A, then T(x) is a yes input for B
    - if x is a no input for A, then T(x) is a no input for B.
    - T can be computed in polynomially bounded time.

“Consequences” of NP-Completeness

- NP-Complete the set of the“hardest” problems in NP, with these important properties:
  - If any *one* NP-Complete problem can be solved in polynomial time…
  - …then *every* NP-Complete problem can be solved in polynomial time…
  - …and in fact *every* problem in **NP** can be solved in polynomial time (which would show **P = NP**)
  - Or, prove an exponential lower-bound for *any single* **NP-C** problem, then *every* **NP-C** problem is exponential

Therefore: solve (say) traveling salesperson problem in O(n^{100}) time, you've proved that **P = NP**. Retire rich & famous!

What We <u>Don't</u> Know: Open Questions

- Is it **impossible** to solve an NP-c problem in polynomial time?
  - No one has proved an exponential lower bound for any problem in NP.
  - But, most computer scientists <u>believe</u> such a lower bound exists for NP-c problems.
- Are all problems in NP tractable or intractable?
  I.e., does P=NP or not?
  - If someone found a polynomial solution to any NP-c problem, we'd know P = NP.
  - But, most computer scientists <u>believe</u> P≠ NP.