DSA2 Final Exam Part 2

Name

For this exam, you should answer each question and compile all of your responses into a **pdf** document. This pdf will be uploaded to Gradescope before the deadline. You have 24 hours to complete this exam. The deadline is Friday (5/8) at 5pm Eastern Time.

There are 4 pages to this exam.

This exam is open textbook, notes, calculator, etc. However, it is **CLOSED** friends, TAs, instructor, etc. Please post on Piazza or email course staff if you have clarification questions on the exam. Good luck!

In theory, there is no difference between theory and practice.
But, in practice, there is.

Page 2: Divide and Conquer

1. [6 points] Use the substitution method to show that $T(n) = 16T(\frac{n}{4}) + n \in O(n^2)$.

2. [6 points] Given an array of integers (e.g., $A = \{3, -4, 5, -2, -2, 6, -3, 5, -3, 2\}$), find the contiguous range of numbers that leads to the maximum sum. For example, the array example given can be summed from index 2 to 7 (the two 5's and everything in between) for a sum of $5 + -2 + -2 + 6 + -3 + 5 = 9$. Solve this problem using divide and conquer in $\Theta(nlogn)$ time. Your recurrence should be $T(n) = 2T(\frac{n}{2}) + n$

Page 3: Dynamic Programming

You build houses, and purchased a street with n empty plots of land. You know how to build three styles of home (Victorian, Cape Cod, and Craftsman) and your customers DO NOT want their neighbor (on either side) to have the same style. Find the best way to build on these plots such that no two neighboring houses are the same style while maximizing profit!

The input will contain three arrays of size n. Each array will specify the price you can sell a house of the given style on plot i (divided by 10,000). For example, if $V = \{20, 14, 35\}$, $CC = \{26, 10, 10\}$ and $CR = \{3, 8, 19\}$, then you can optimize your profit by selling a Cape Cod on plot 1 ($CC[1] = 26$), a Craftsman on plot 2 ($CR[2] = 8$), and a Victorian on plot 3 ($V[3] = 35$). Your total profit would then be $CC[1] + CR[2] + V[3] = 26 + 8 + 35 = 69 * 10,000 = \$690,000$.

3. [3 points] Suppose our sub-problem definition is $P(i, s)$, representing the optimal profit of building on the first i plots and building style s home on the last (ith) plot. State the base cases.

4. [6 points] State a recursive solution to $P(i, s)$.

5. [3 points] State the runtime of your dynamic programming solution.

Page 4: Reductions

Suppose we want to solve a variation of the **max-flow** problem. In this variant, you are still given a *flow network* as input, and you are still attempting to maximize the amount of flow moving through the network. However, the difference this time is that for a given edge with capacity c, you MUST push exactly 0 flow, or c flow through that edge. In other words, every edge in the network must be completely full to capacity or at 0 flow. Let's call this the *0/1 Max-Flow Problem* because you must push 0 (none) or 1 (all) of the flow through each edge. On this page, you will show that the *0/1 Max-Flow Problem* is NP-Complete.

6. [6 points] First, show that the *0/1 Max-Flow Problem* is in *NP*. Provide a verification algorithm that given a flow amount for each edge, verifies if that flow is valid and exceeds a given value k. Briefly describe your algorithm.

7. [6 points] Now, show that this problem is *NP-HARD* by providing a reduction from *Independent Set*. Recall that *Independent Set* states that if given a graph, and an integer k, can you find a set of k nodes such that no two of the chosen nodes are adjacent. Describe your redeuction.