DSA2 Final Exam Part 1

Name ______________________________

For this exam, you should answer each question and compile all of your responses into a **pdf** document. This pdf will be uploaded to Gradescope before the deadline. You have 24 hours to complete this exam. The deadline is Friday (5/8) at 5pm Eastern Time.

There are 4 pages to this exam.

This exam is open textbook, notes, calculator, etc. However, it is **CLOSED** friends, TAs, instructor, etc. Please post on Piazza or email course staff if you have clarification questions on the exam. Good luck!

In theory, there is no difference between theory and practice.
But, in practice, there is.

Page 2: Graphs

1. [6 points] You are navigating a cavern full of one way tunnels (each is a downward slide, and you can't climb back!). The cavern has junction points where you can slide down one of several next tunnels. The cavern has one exit, no cycles, many unique paths to the one exit, and many paths that lead to dead ends. Let's call each junction point j_i and let o_i be the number of outgoing tunnels from j_i. At each junction, you choose a tunnel uniformly at random ($\frac{1}{o_i}$ chance each). Describe an algorithm that returns the junction you are most likely to get stuck at. *Remember the chance of going down a path is the product of the chance you take each edge along that path, and the chance of ending up at a junction is the sum of the chance you come in from each unique incoming edge.*

2. [6 points] Watch this youtube video. Once you have watched, summarize the main points of the video in a couple of sentences. Then, describe what would happen if your sliding puzzle assignment on Gradescope runs on one of the bad configurations from the video. What would happen? *If link doesn't work, direct url is https://www.youtube.com/watch?v=YI1WqYKHi78*

Page 3: Flow Networks

3. [6 points] Consider a flow network in which vertices, as well as edges, have capacities. That is, the total positive flow entering any given vertex is subject to a capacity constraint. Show that determining the maximum flow in a network with edge and vertex capacities can be reduced to an ordinary maximum-flow problem on a flow network of comparable size.

4. [6 points] The **edge connectivity** of an undirected graph is the minimum number k of edges that must be removed to disconnect the graph. For example, the edge connectivity of a tree is 1, and the edge connectivity of a cyclic chain of vertices is 2. Briefly describe how to determine the **edge connectivity** of an undirected graph $G = (V, E)$ by running a maximum-flow algorithm on at most $|V|$ flow networks, each having $O(V)$ vertices and $O(E)$ edges.

Page 4: Greedy Algorithms

Suppose you are given two sets A and B, each containing n positive integers. You can choose to reorder each set however you like. After reordering, let a_i be the ith element of set A, and let b_i be the ith element of set B. You then receive a payoff of $\prod_{i=1}^{n} a_i^{b_i}$. On this page, you will provide a solution to this problem.

5. [4 points] Show that this problem has optimal substructure. Specifically, if an optimal solution to the whole problem is to order the sets $A = (a_1, a_2, ..., a_n)$ and $B = (b_1, b_2, ..., b_n)$, then the sets $A' = (a_2, ..., a_n)$ and $B' = (b_2, ..., b_n)$ is the optimal solution to the sub-problem that ignores a_1 and b_1.

6. [4 points] Provide a greedy choice that solves this problem.

7. [4 points] In a couple of sentences, argue, using an exchange argument, that the greedy choice leads to an optimal solution. This answer needs to be logically sound, but does not need to be formal.