jh2jf

Collaboration Policy: You are encouraged to collaborate with up to 4 other students, but all work submitted must be your own *independently* written solution. List the computing ids of all of your collaborators in the `collabs` command at the top of the tex file. Do not share written notes, documents (including Google docs, Overleaf docs, discussion notes, PDFs), or code. Do not seek published or online solutions for any assignments. If you use any published or online resources (which may not include solutions) when completing this assignment, be sure to cite them. Do not submit a solution that you are unable to explain orally to a member of the course staff. Any solutions that share similar text/code will be considered in breach of this policy. Please refer to the syllabus for a complete description of the collaboration policy.

Collaborators: Tom Horton (tbh3f)

Sources: Cormen, et al, Introduction to Algorithms; Adams, Douglas, Hitchhiker's Guide to the Galaxy

PROBLEM 1 *Writing Math in LaTeX*

The main reason for using LaTeX this semester is to present math more neatly and clearly. There are two main ways to include math in your documents. The first is inline math, which you use when you want to include math among regular English text. This is done by putting your math between $ symbols. For example, the statement "if $x \in \mathbb{N}$ then $S \neq \emptyset$" is produced using inline text. For each line below, add on the mathematical symbol/expression we've described using inline text. The first two are done for you.

- The symbol for set membership: $\in$
- The fraction one half: $\frac{1}{2}$
- The expression square root of 2: $\sqrt{2}$
- The fraction 1 divided by the square root of 2: $\frac{1}{\sqrt{2}}$
- The mathematical symbol pi: π
- The expression "S is a subset of the real numbers": $S \subseteq \mathbb{R}$
- The expression "the empty set is a proper subset of the rational numbers": $\emptyset \subset \mathbb{Q}$

PROBLEM 2 *Proofs*

Learn how to typeset math and construct proofs by reproducing the second proof below. You will need to use the `eqnarray` or `align` environment, as well as the `eqnarray*` or `align*` environment. Note the reference in red, which should refer correctly to the equation (look up the `ref` command). The first proof is provided as an example.

Definition 1 *A rational number is a fraction $\frac{a}{b}$ where a and b are integers.*

Theorem 1 *$\sqrt{2}$ is irrational.*

Proof. By Contradiction. For a rational number $\frac{a}{b}$, without loss of generality we may suppose that a and b are integers which share no common factors, as otherwise we could remove any common factors (i.e. suppose $\frac{a}{b}$ is in simplest terms). To say $\sqrt{2}$ is irrational is equivalent to stating that 2 cannot be expressed in the form $(\frac{a}{b})^2$. Equivalently, this says that there are no integer values for a and b satisfying

$$a^2 = 2b^2 \tag{1}$$

Assume toward reaching a contradiction that Equation 1 holds for a and b being integers without any common factor between them. It must be that a^2 is even, since $2b^2$ is divisible by 2, therefore a is even. If a is even, then for some integer c

$$\begin{aligned} a &= 2c \\ a^2 &= (2c)^2 \\ 2b^2 &= 4c^2 \\ b^2 &= 2c^2 \end{aligned}$$

therefore, b is even. This implies that a and b are both even, and thus share a common factor of 2. This contradicts our hypothesis, therefore our hypothesis is false. □

Theorem 2 *If $n \in \mathbb{Z}$ is a non-prime integer with $n > 1$, then $2^n - 1$ is not prime [from Velleman, How to Prove It: A Structured Approach, 2006].*

Proof. Direct Proof. Since n is not prime, $\exists a, b \in \mathbb{Z}$ such that $a < n$ and $b < n$ and $n = ab$. Let

$$x = 2^b - 1$$

and

$$y = 1 + 2^b + 2^{2b} + ... + 2^{(a-1)b}.$$

Then,

$$\begin{aligned} xy &= (2^b - 1)(1 + 2^b + ... + 2^{(a-1)b}) & (2) \\ &= 2^b(1 + 2^b + ... + 2^{(a-1)b}) - (1 + 2^b + ... + 2^{(a-1)b}) & (3) \\ &= 2^{ab} - 1 & (4) \\ &= 2^n - 1. & (5) \end{aligned}$$

Since $b < n$, then $x = 2^b - 1 < 2^n - 1$. Likewise, since $ab = n > a$, we know that $b > 1$ and $x = 2^b - 1 > 2 - 1 = 1$. Therefore, $y < xy = 2^n - 1$ and $2^n - 1$ can be written as the multiplication of x and y by Equation 5. Therefore $2^n - 1$ is not prime. □

PROBLEM 3 *Passages*

Include a passage from **your** favorite book using the `quote` environment. Cite your source in `sources` at the top of this file.

> "Forty-two!" yelled Loonquawl. "Is that all you've got to show for seven and a half million years' work?"
> "I checked it very thoroughly," said the computer, "and that quite definitely is the answer. I think the problem, to be quite honest with you, is that you've never actually known what the question is." [Adams, Douglas. *Hitchhiker's Guide to the Galaxy*. 1980]

PROBLEM 4 *Sketchings*

Learn how to include drawings in your documents with the `\includegraphics{file}` command by submitting a caricature of Professor Horton or Professor Hott.